Journées de Modélisation des Vagues à Phases Résolues
Ile d'Aix, 4 - 6 octobre 2023

Surf eddies and mixing in 3D wave-resolving models

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Motivation

A need to assess the fate of coastal pollutants, sediments, ecosystems, people …

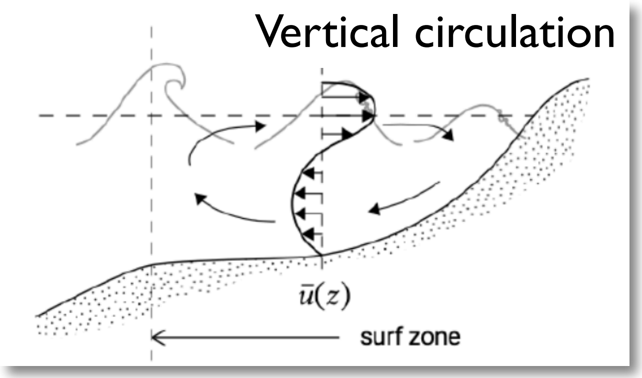
Motivation

coastal pollutants …

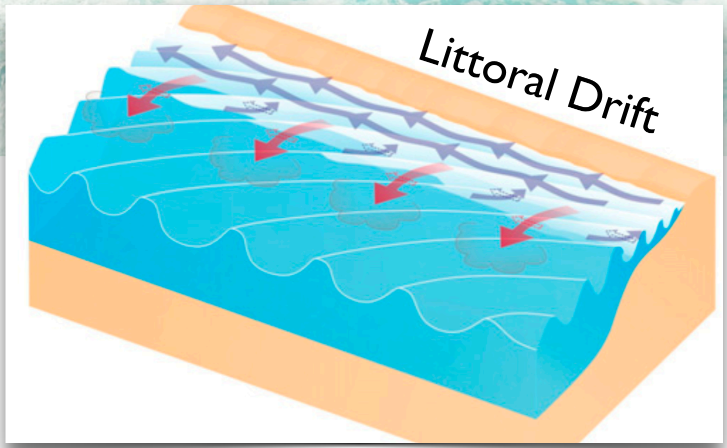
Phosphates liquid waste discharged into the coastal waters without treatment (photo: A. Blivi, Togo 2006)

Stationary and transient circulation

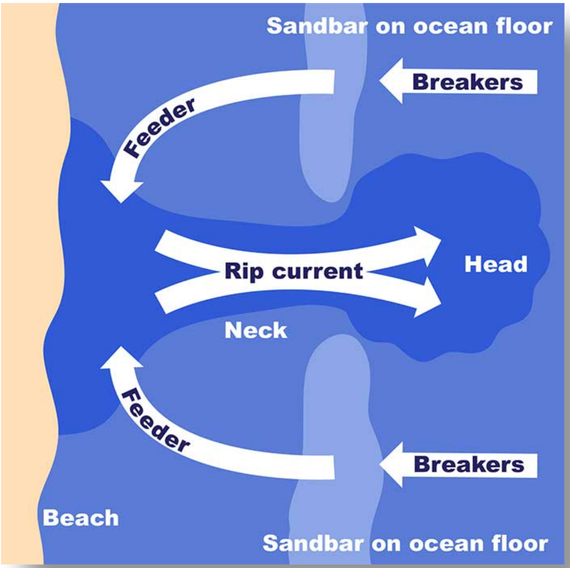
Vertical circulation



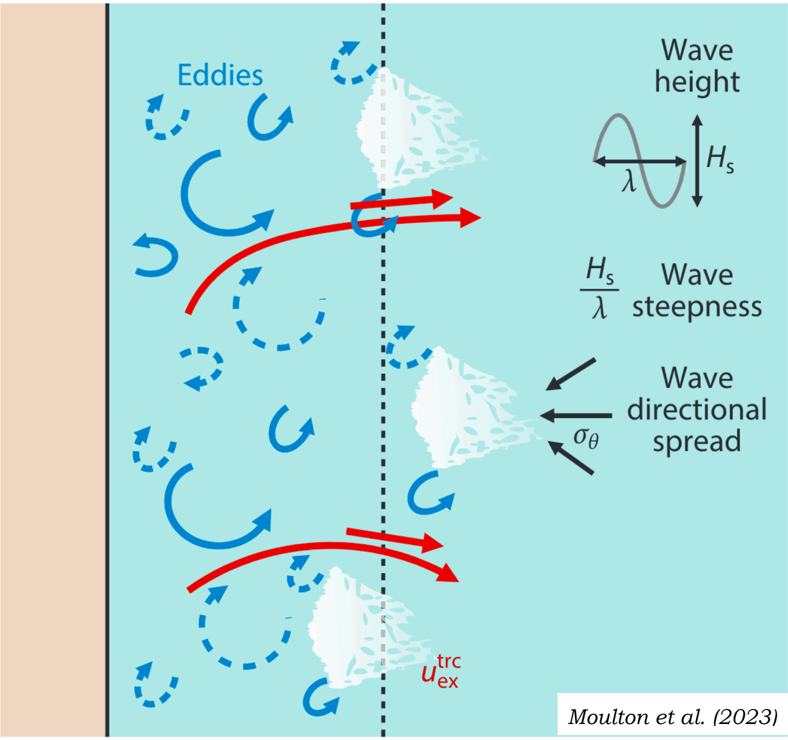
Littoral Drift

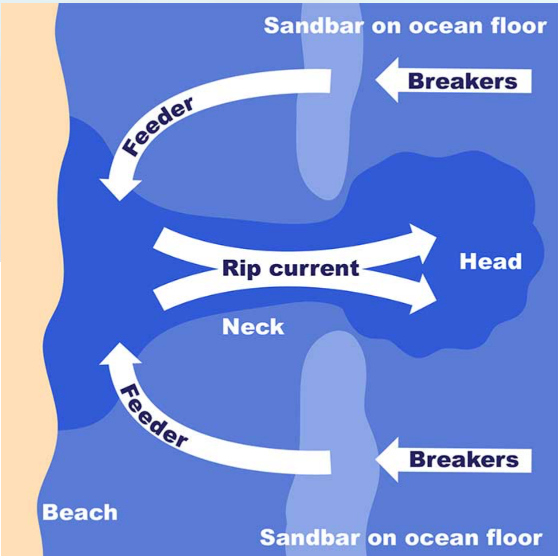


Channeled Rip Currents



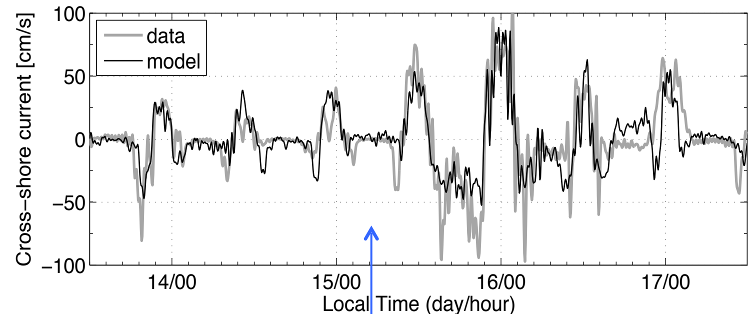
Flash rips from short-crested waves

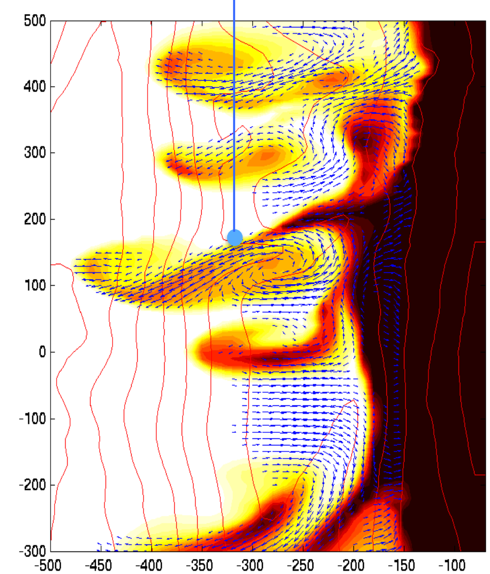




3D wave-averaged CROCO
Channeled rip currents

Biscarrosse Beach, June 2007
(Marchesiello et al., 2015)





Wave-averaged equations (McWilliams et al., 2004):

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla_\perp)\mathbf{u} + w\frac{\partial \mathbf{u}}{\partial z} + f\hat{\mathbf{z}}\times\mathbf{u} + \nabla_\perp\phi - \mathbf{F} = -\nabla_\perp\mathscr{K} + \mathbf{J} + \mathbf{F}^w,$$

$$\frac{\partial \phi}{\partial z} + \frac{g\rho}{\rho_0} = -\frac{\partial \mathscr{K}}{\partial z} + K,$$

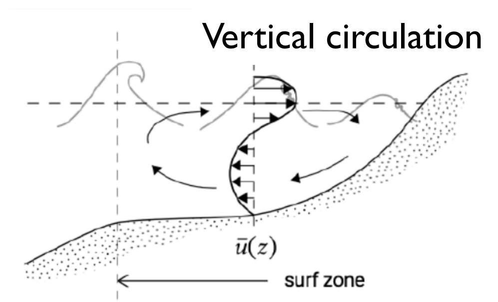
$$\nabla_\perp\cdot\mathbf{u} + \frac{\partial w}{\partial z} = 0,$$

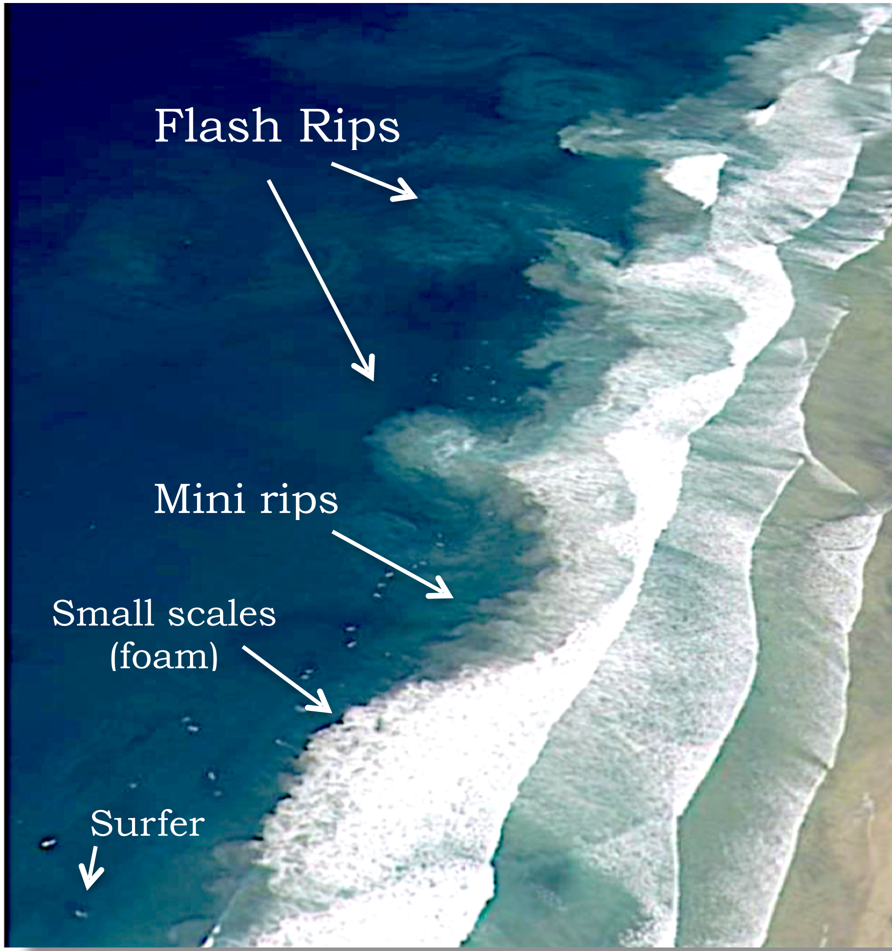
$$\frac{\partial c}{\partial t} + (\mathbf{u}\cdot\nabla_\perp)c + w\frac{\partial c}{\partial z} - \mathscr{C} = -(\mathbf{u}^{St}\cdot\nabla_\perp)c - w^{St}\frac{\partial c}{\partial z} + \frac{1}{2}\frac{\partial}{\partial z}\left[\mathscr{E}\frac{\partial c}{\partial z}\right].$$

What about transient rips ?

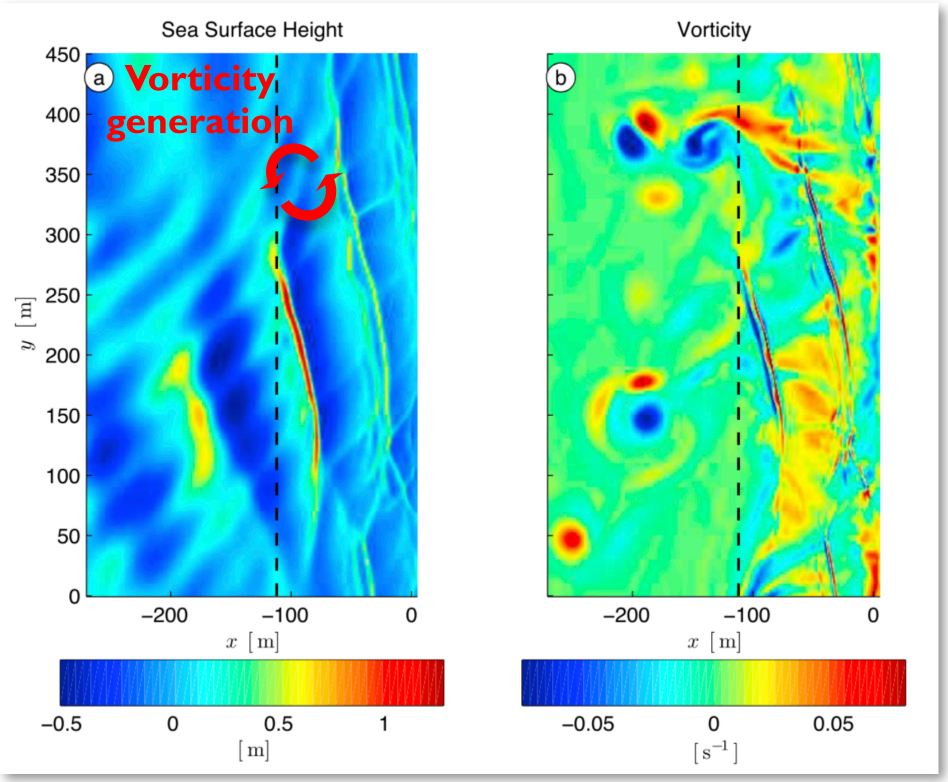
- ✓ The nearshore zone is **essentially non-stationary** (Tang & Dalrymple, 1989):
  - need for wave-resolving models

- ✓ Surfzone eddies affected by **vertical shear** (Marchesiello et al., 2021):
  - need for **3D** wave-resolving models



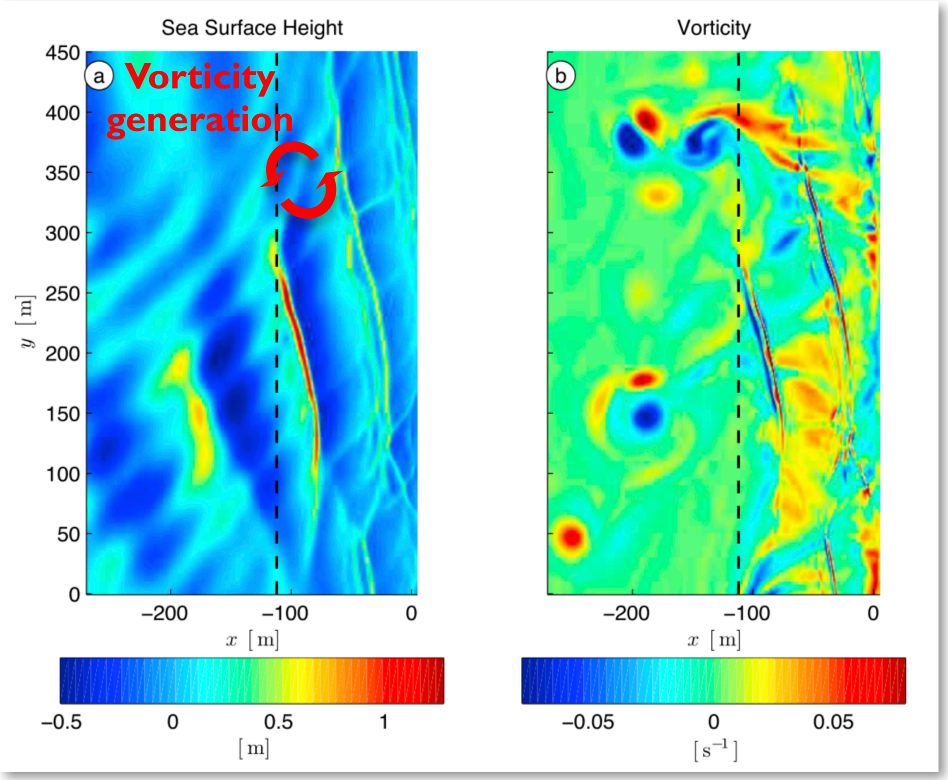


Flash rips in 2D Boussinesq wave-resolving models



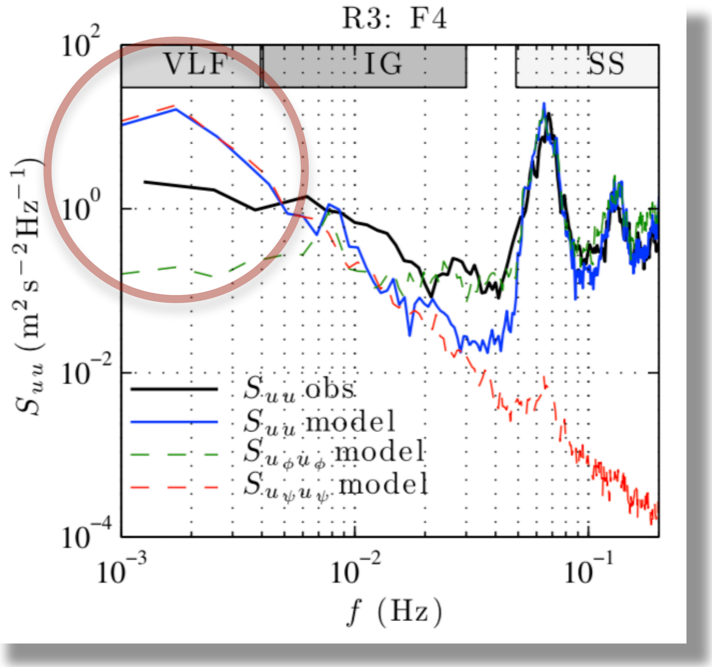
2D wave-resolving Boussinesq model
(Feddersen et al., 2011)

Flash rips in 2D Boussinesq wave-resolving models



2D wave-resolving Boussinesq model
(Feddersen et al., 2011)

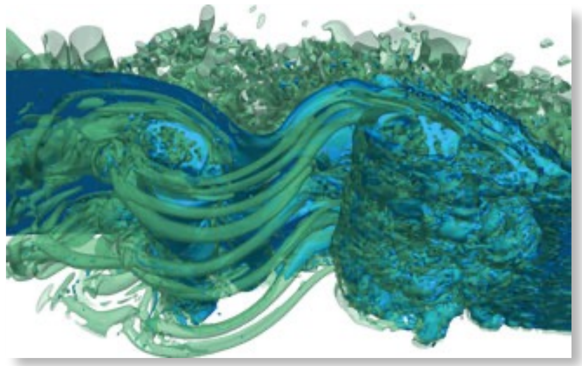
Too much VLF energy?



Feddersen et al. (2011)
Also Spydell & Feddersen (2009)
Kirby & Derakhti (2019)

3D wave-resolving models

- VOF/SPH LES models: “Direct” turbulence

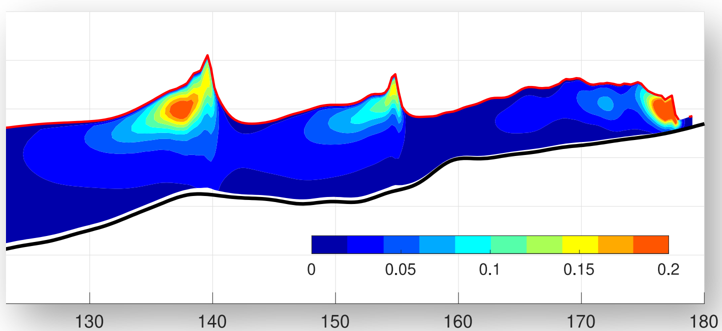


(Lubin & Glockner, 2015)

Time scale < wave period

- Free-surface RANS models:

CROCO
NHWAVE
SWASH



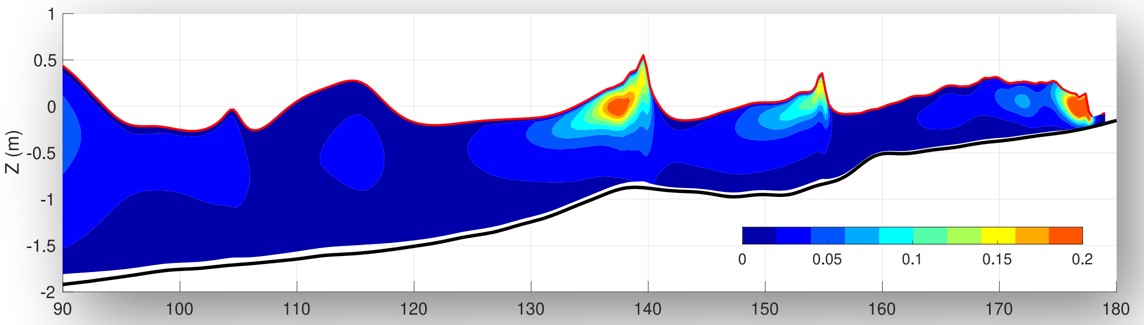
(Marchesiello et al., 2021)

Time scale > wave period

A turbulence model is needed for time scales < wave period

CROCO

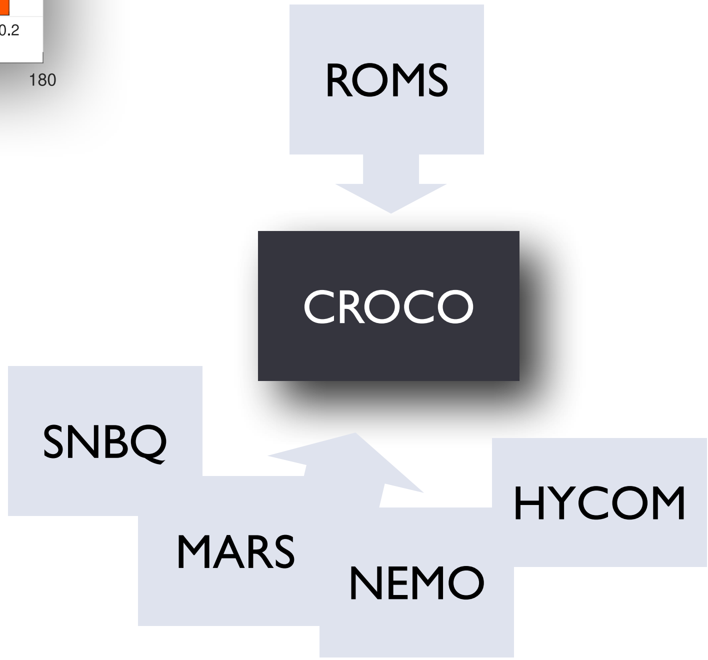
Coastal and Regional Ocean COmmunity model



CROCO project:

- ✓ Accuracy & efficiency
- ✓ Realistic applications
- ✓ Large user community

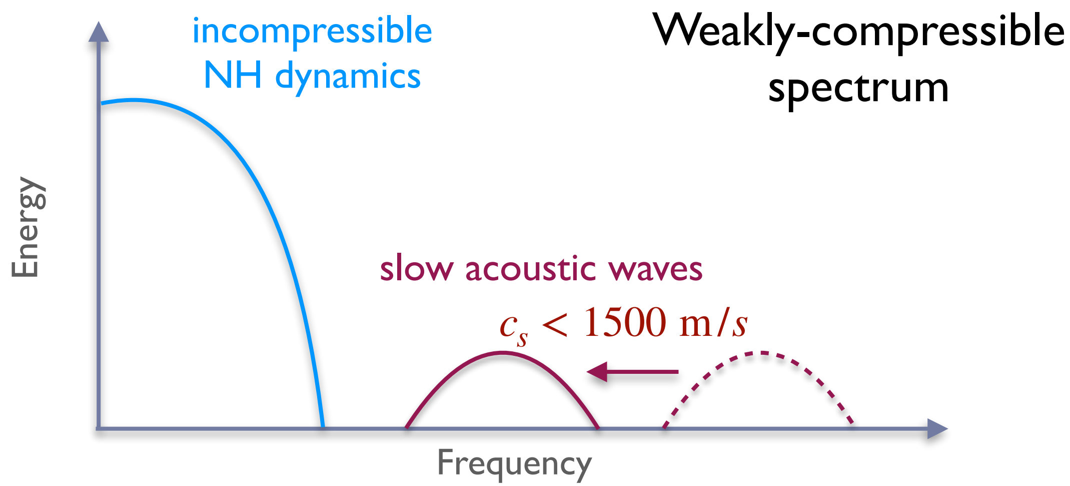
CONSORTIUM



Non-hydrostatic solver

Non-hydrostatic solver

- Weakly compressible approach (Auclair et al., 2018)



In the nearshore, where long waves are slow, we can use a sound speed of order 10 m/s

Non-hydrostatic solver

- Weakly compressible approach

$$p = p_a + p_H + c_s^2 \rho_f \quad \longleftarrow \quad P_{NH}$$

Homogeneous linearized equations

$$\partial_t u = -g\partial_x \eta - c_s^2 \partial_x \rho_f / \rho_0$$

$$\partial_t w = -c_s^2 \partial_z \rho_f / \rho_0$$

$$\partial_t \rho_f = -\rho_0 \left(\partial_x u + \partial_z w\right)$$

Boundaty conditions

$$\rho_f|_{z=\eta} = 0$$

$$\partial_t \eta = w|_{z=0}$$

$$w|_{z=-H} = 0$$

Local NH pressure correction rather than global correction through elliptic solver:

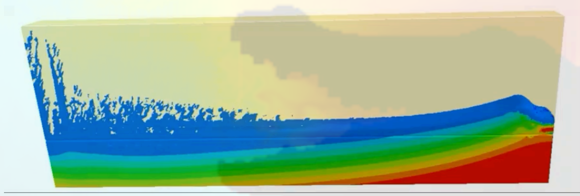
- **better parallel performances**
- **Exact boundary conditions**

Efficiency :

- Small c_s ($\gtrsim \sqrt{gh}$)
- Time-splitting

Chen et al. (2003)

CROCO test cases: TANK

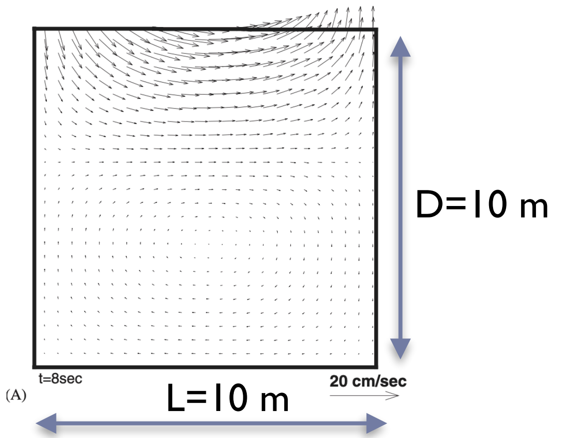


Standing wave caused by a sinusoidal free-surface set-up

$$\eta_i = a \cos kx$$

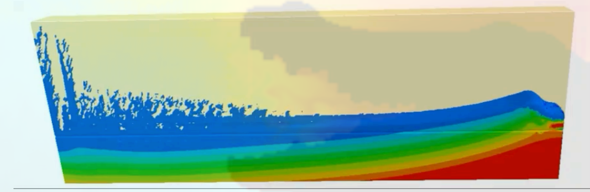
$$a = 1\ mm$$

$$k = \pi / L$$



Sloshing test cases:

Chen et al. (2003)



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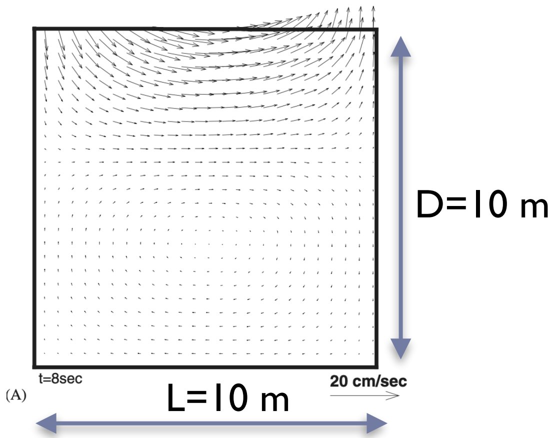
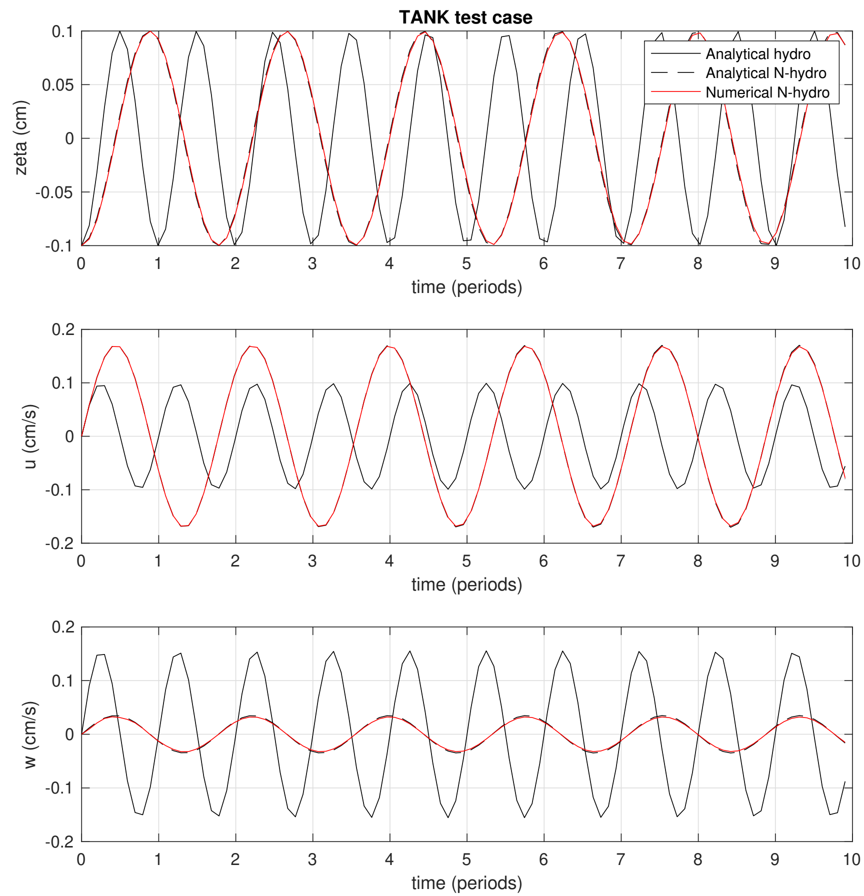


NH Waves

$T \sim 3.6\ s$

$$\begin{cases} \sigma = \sqrt{gk \tanh kD} \\ \eta = a \cos kx \cos \sigma t \\ u = a\sigma \dfrac{\sin \sigma t}{\sinh kD} \sin kx \cosh kz \\ w = -a\sigma \dfrac{\sin \sigma t}{\sinh kD} \cos kx \sinh kz \end{cases}$$

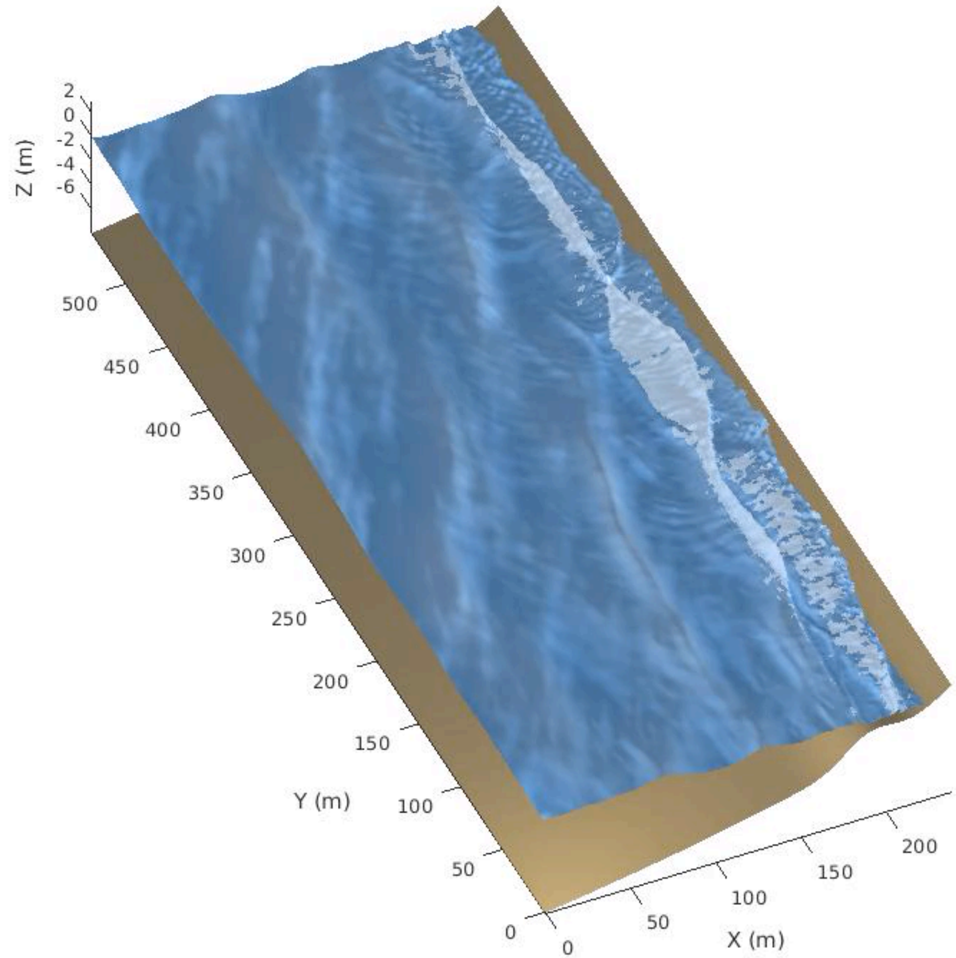
Non-Hydrostatic Case



Wavemaker correction

Wavemaker with frequency and directional spreading

JONSWAP wave spectrum with directional spreading



Wavemaker correction for the standing wave problem

Darlymple (1975), Salatin et al. (2021)

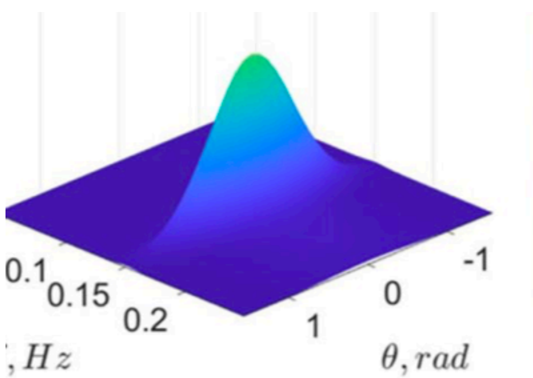
Wave-resolving models generally use a classical double summation wave-maker:

$$\eta_{bc}(y,t) = \sum_i a_i \sum_j d_j cos(k_{y,i,j}y - \omega_i t - \phi_{ij})$$

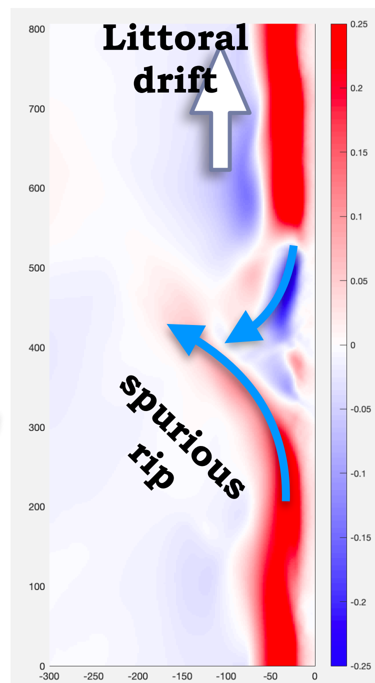
Amplitude depending on a frequency spectrum

Directional spread

… leading to wave coherence, i.e., stationary interferences between waves of different directions with same frequencies (phase-locking)



Wave directional spread



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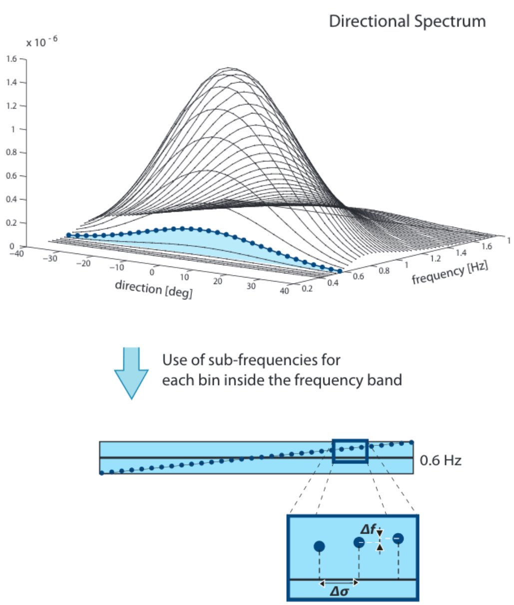
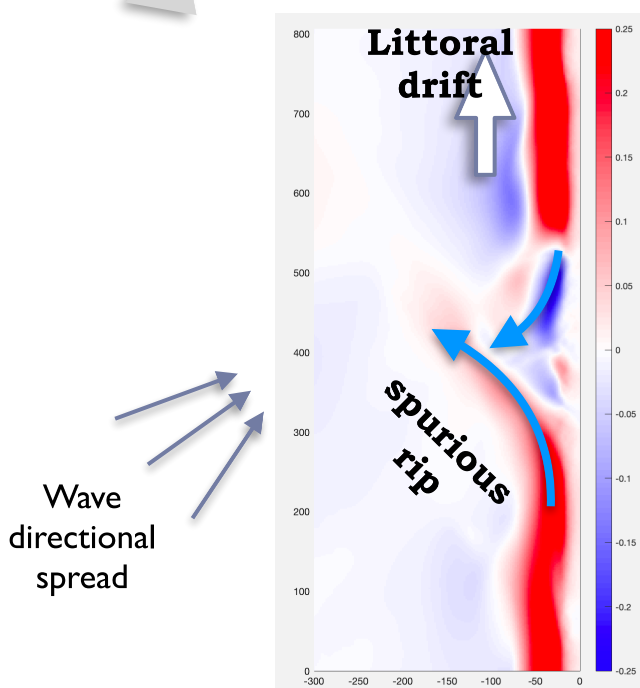
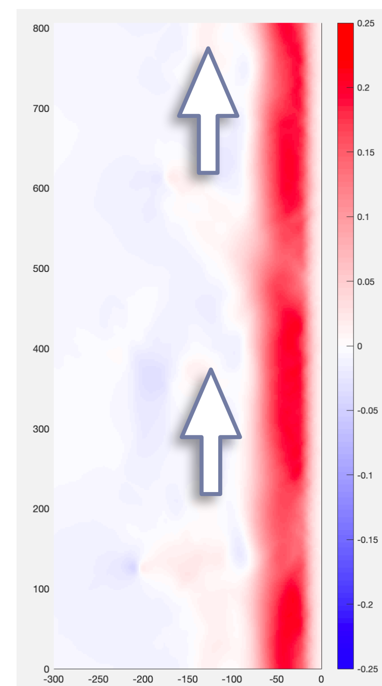


Wave-maker modification such that all wave components have distinct frequencies and directions:

$$\eta_{bc}(y,t) = \sum_i a_i d_i cos(k_{y,i}y - \omega_i t - \phi_i)$$



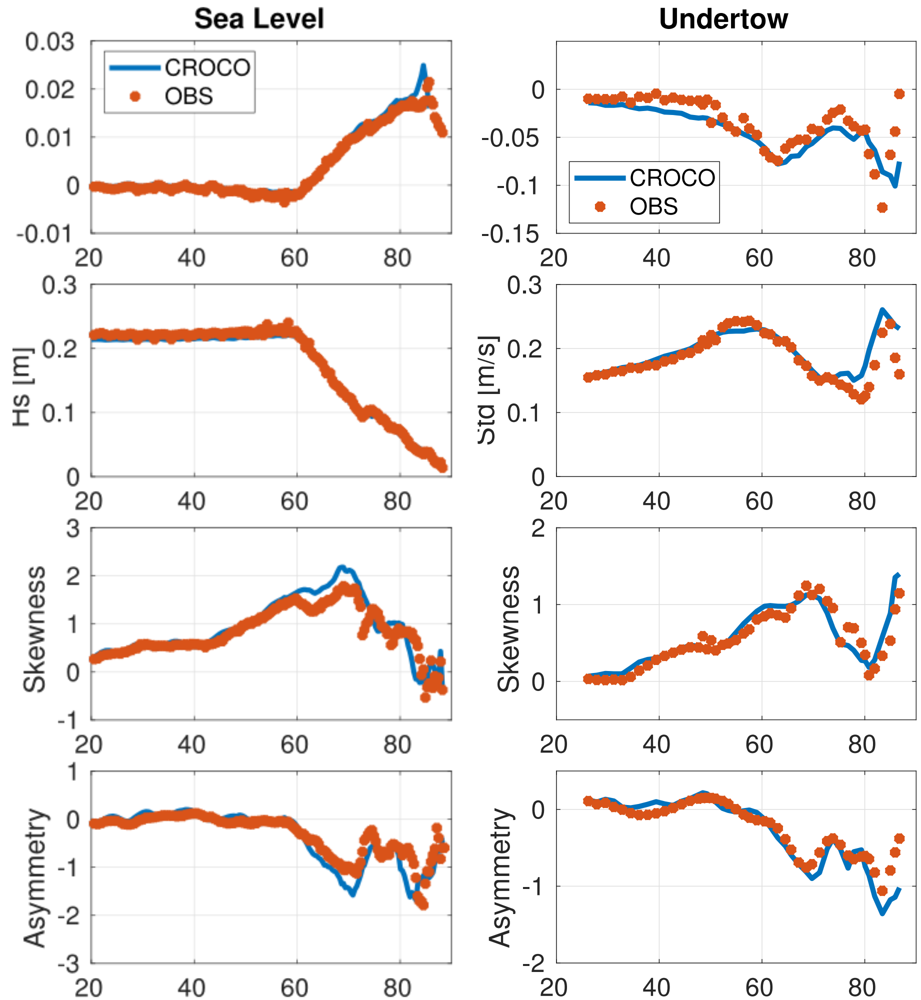


Validation of nearshore waves and currents with laboratory experiments

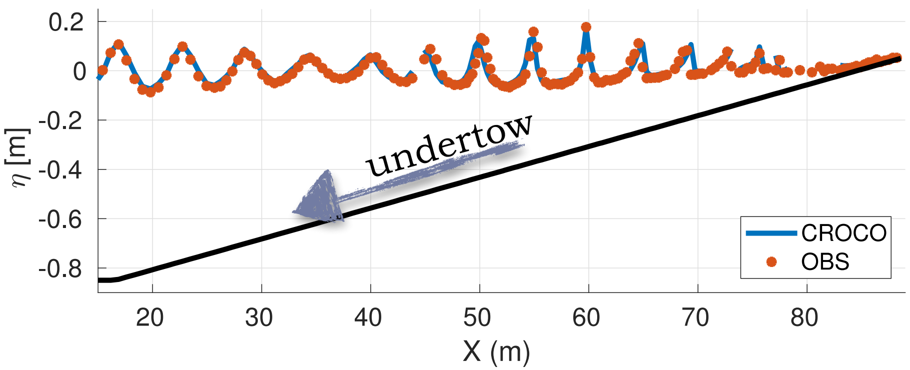
Validation with flume experiments

GLOBEX (B2) - Michalet et al. (2014)

Scheldt Wave Flume (Deltares)



- ✓ Resolution: 6 cm, 10 sigma levels
- ✓ Breaking-induced turbulence: WENO5 + k-ω model



Delta Flume (Deltares)

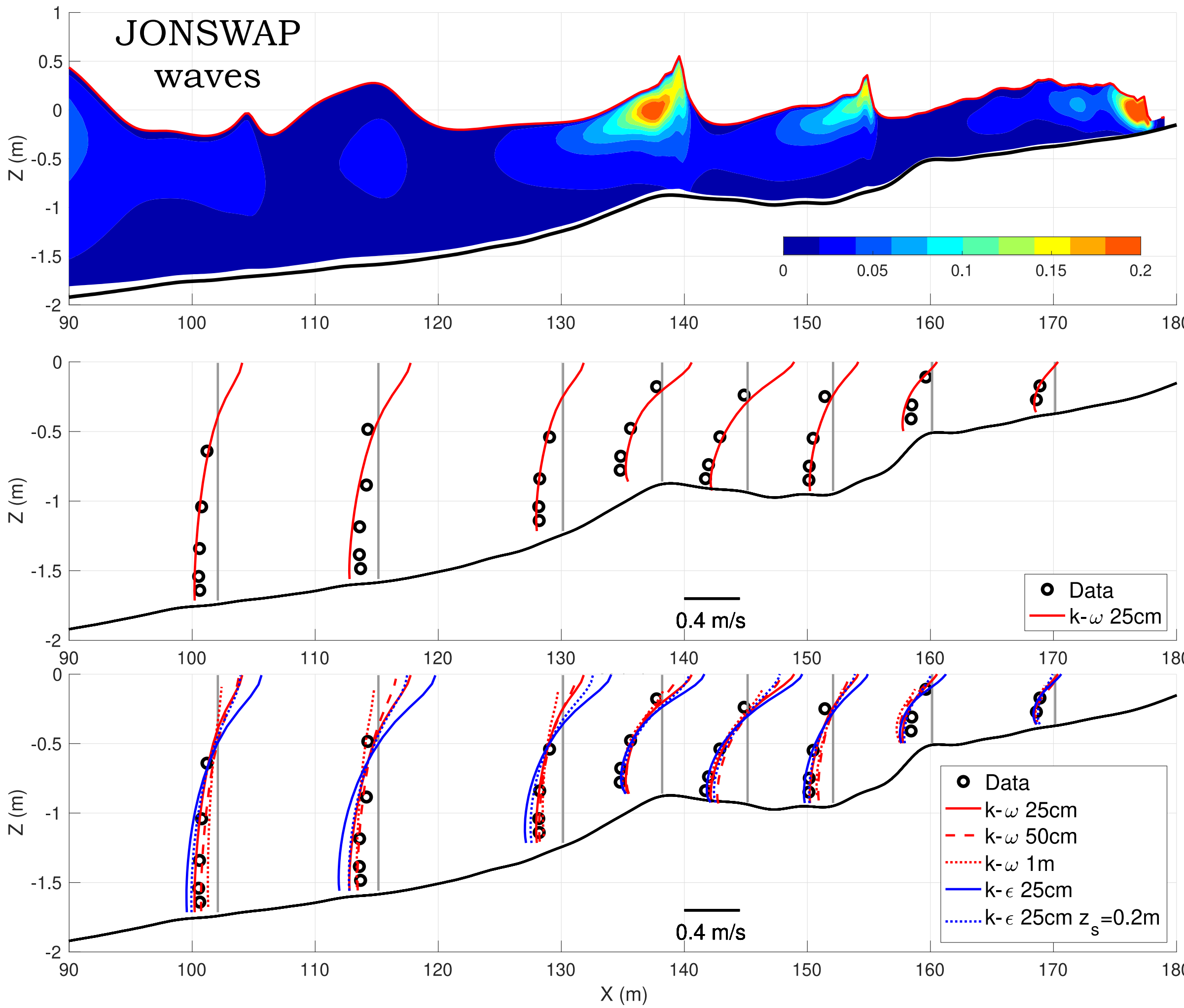
Large-scale flume

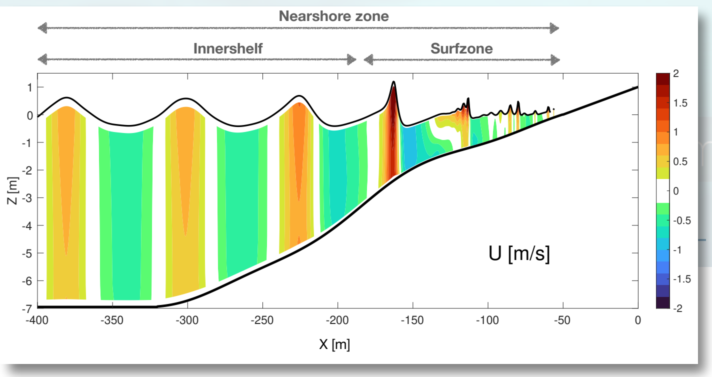
LIP-11D (1B) - Roelvink & Reniers (1995)

TKE

Mean U

Test of turbulence closure





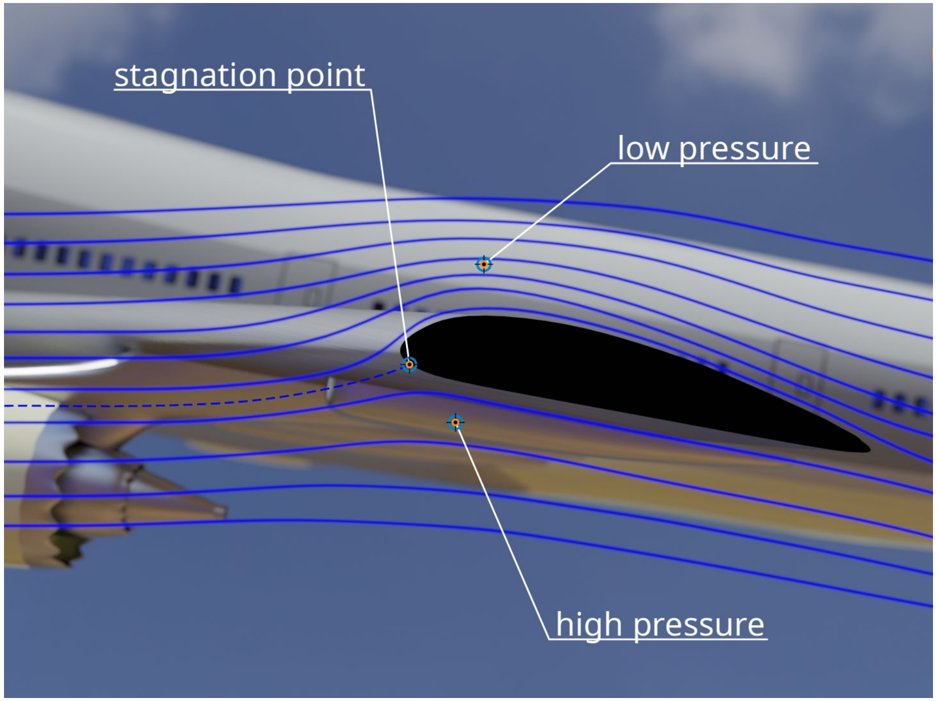
Turbulence model : **overmixing** in potential flow region

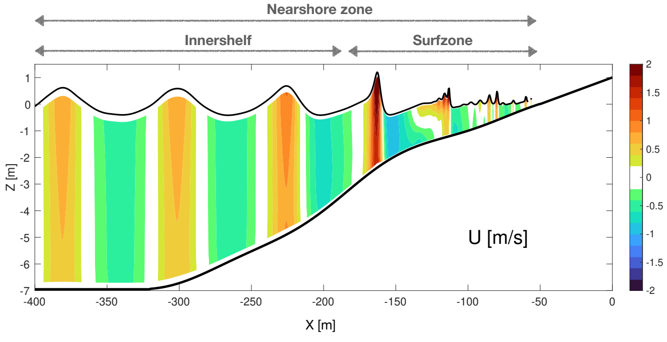
Well-known problem: the **stagnation point anomaly** (Launder and Kato, 1993)

Overmixing when $\Omega \ll S$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \quad \text{Strain}$$

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \quad \text{vorticity}$$





Stabilized turbulence closure

Larsen & Fuhrman (2018),
Marchesiello & Treillou (2023)

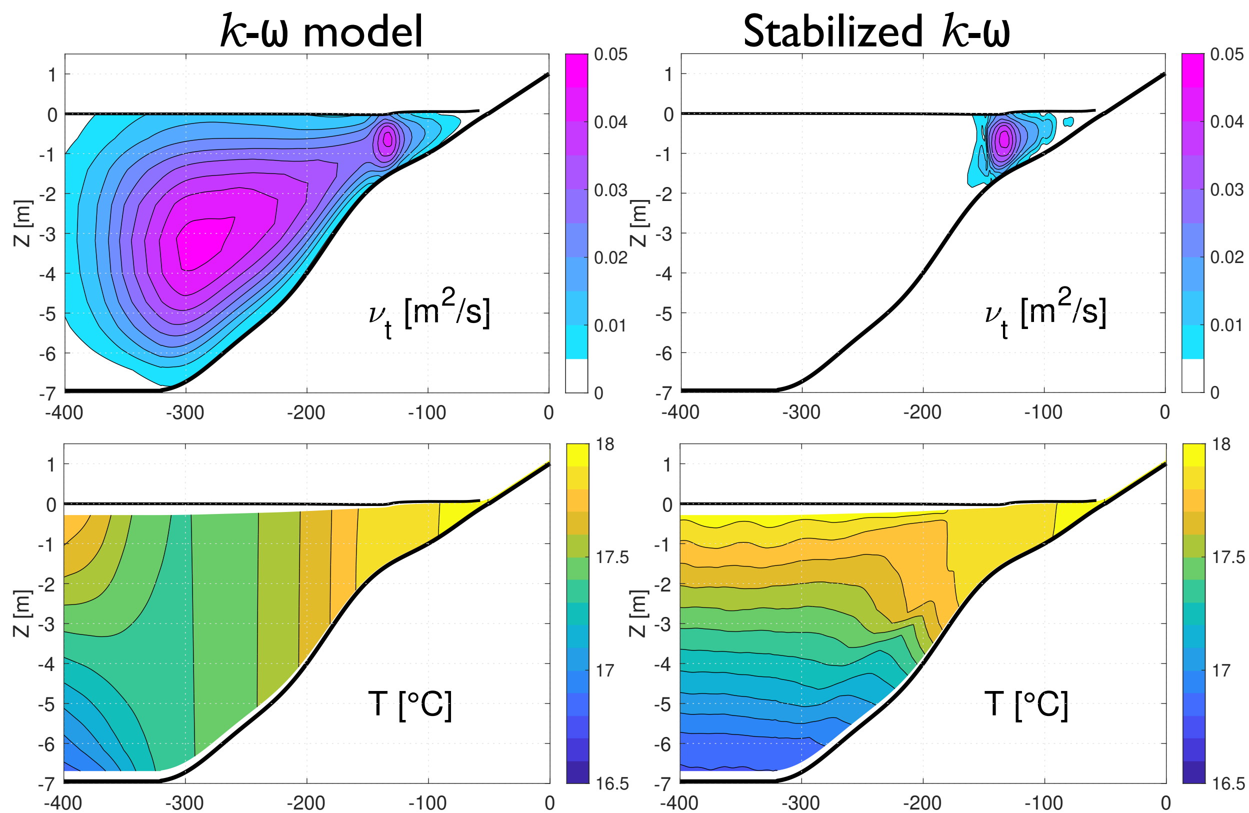
A stabilized turbulence model (limiting P) is needed in potential flow regions to maintain innershelf stratification

$$
\begin{cases}
\dfrac{\partial k}{\partial t} = P - \epsilon \\[2ex]
\dfrac{\partial \omega}{\partial t} = \dfrac{\omega}{k}(c_{\omega 1} P - c_{\omega 2}\epsilon)
\end{cases}
$$

Boussinesq hypothesis

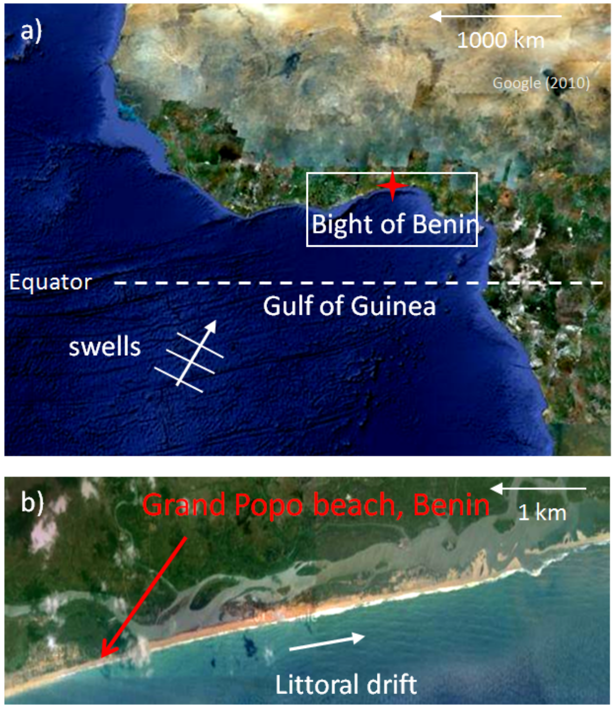
$$
P = -\overline{u_i' u_j'}\frac{\partial u_i}{\partial x_j} = 2\nu_t S^2
$$

limitation when $\Omega / S < \lambda$

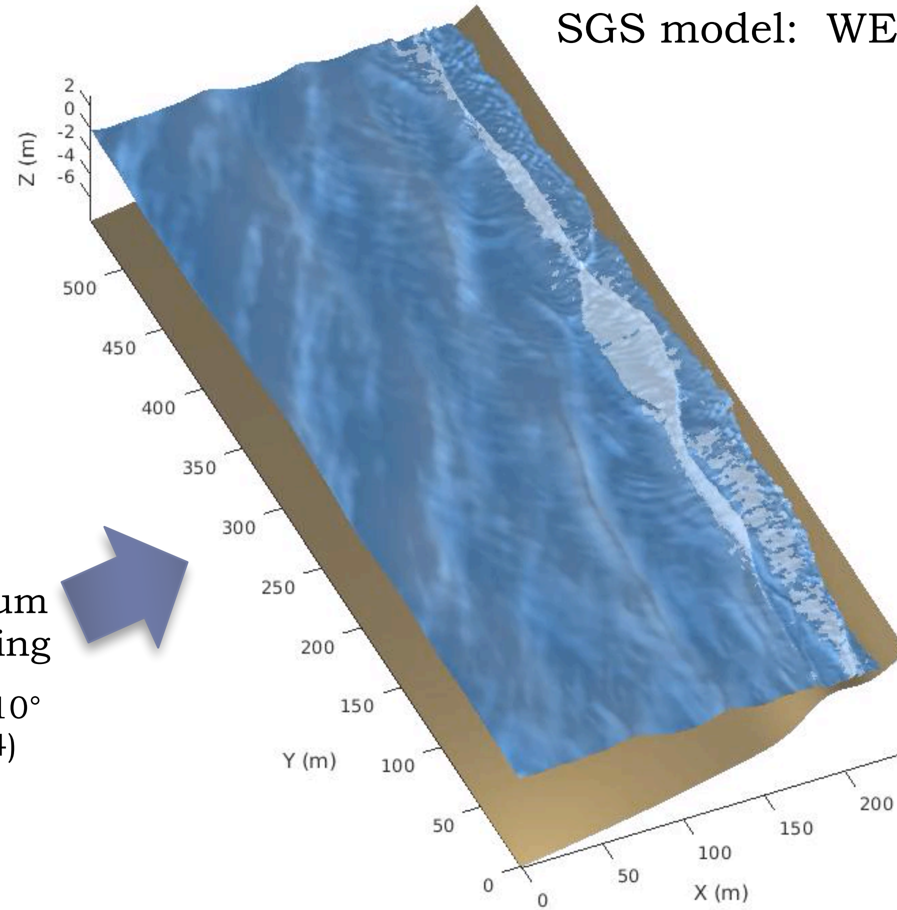


3D application to a natural beach

Application to a longshore-uniform beach in Grand Popo, Benin



Resolution: 50 cm, 10 lev.
SGS model: WENO5 + *k*-ω



JONSWAP wave spectrum
with directional spreading

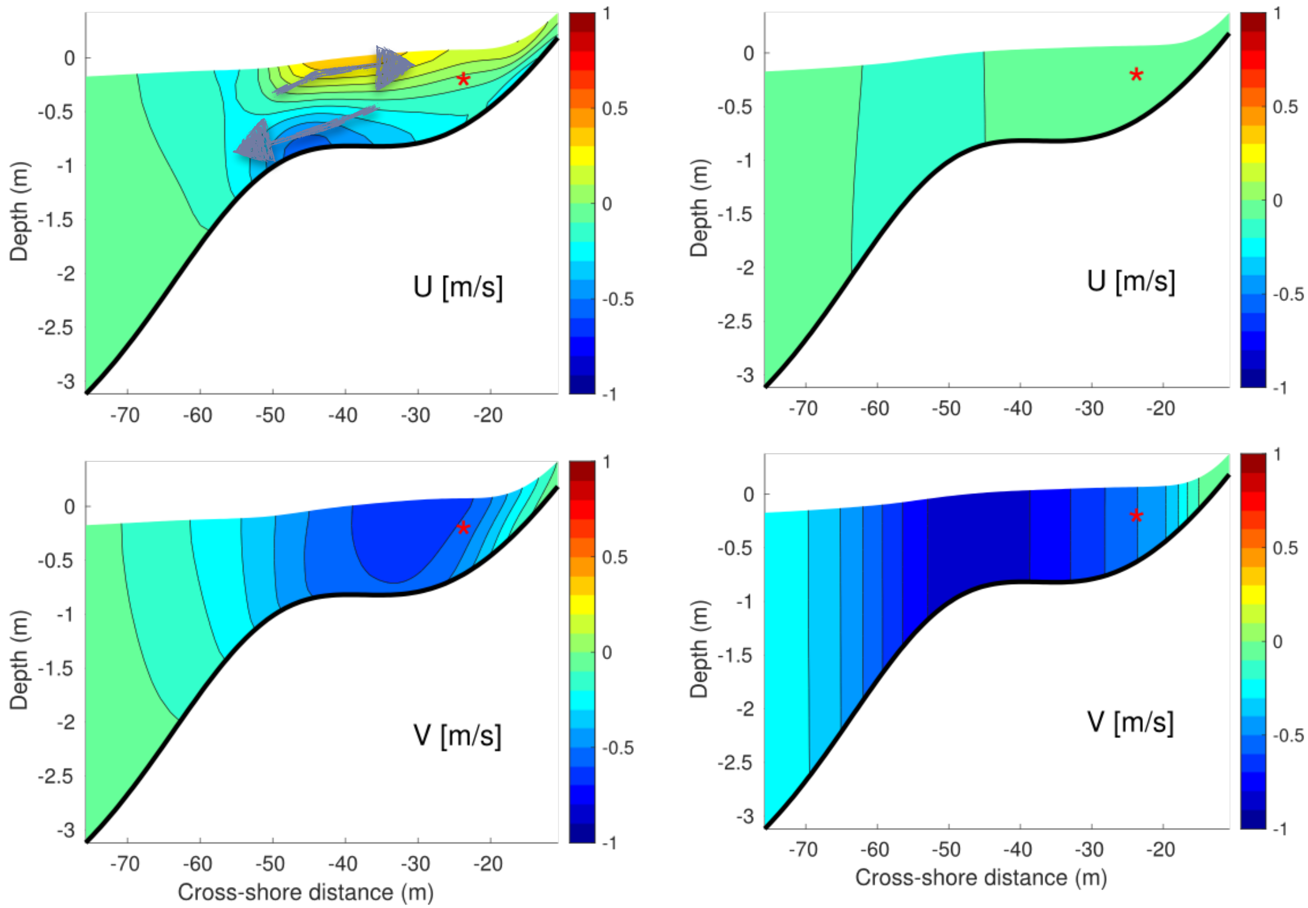
Hs=1.15 m, Tp=11 s, Dir=10°
(mid-tide, March 13 2014)

Shallow vs. Deep breaking experiments

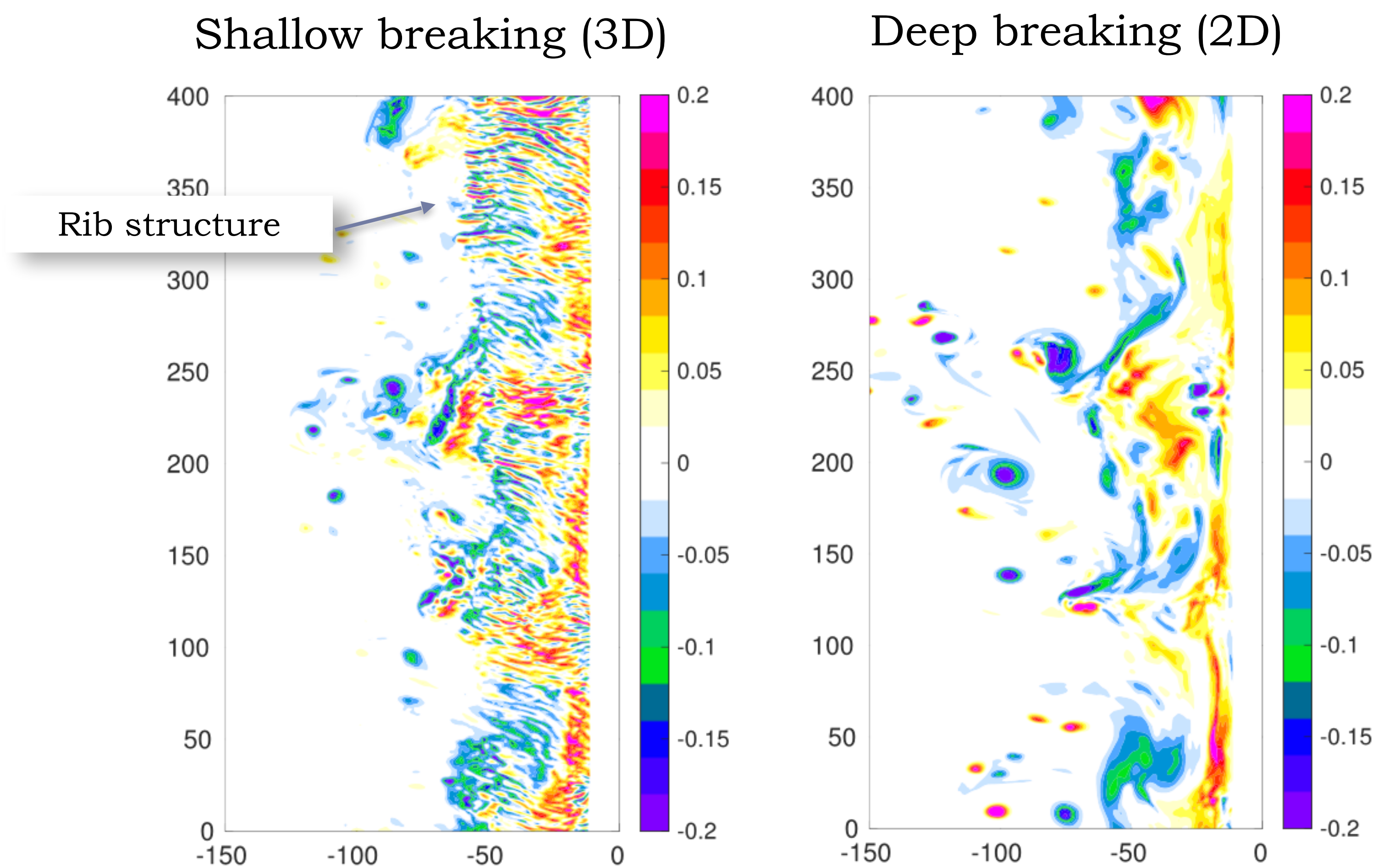
Shallow breaking (3D) | Deep breaking (pseudo-2D)

Cross-shore currents

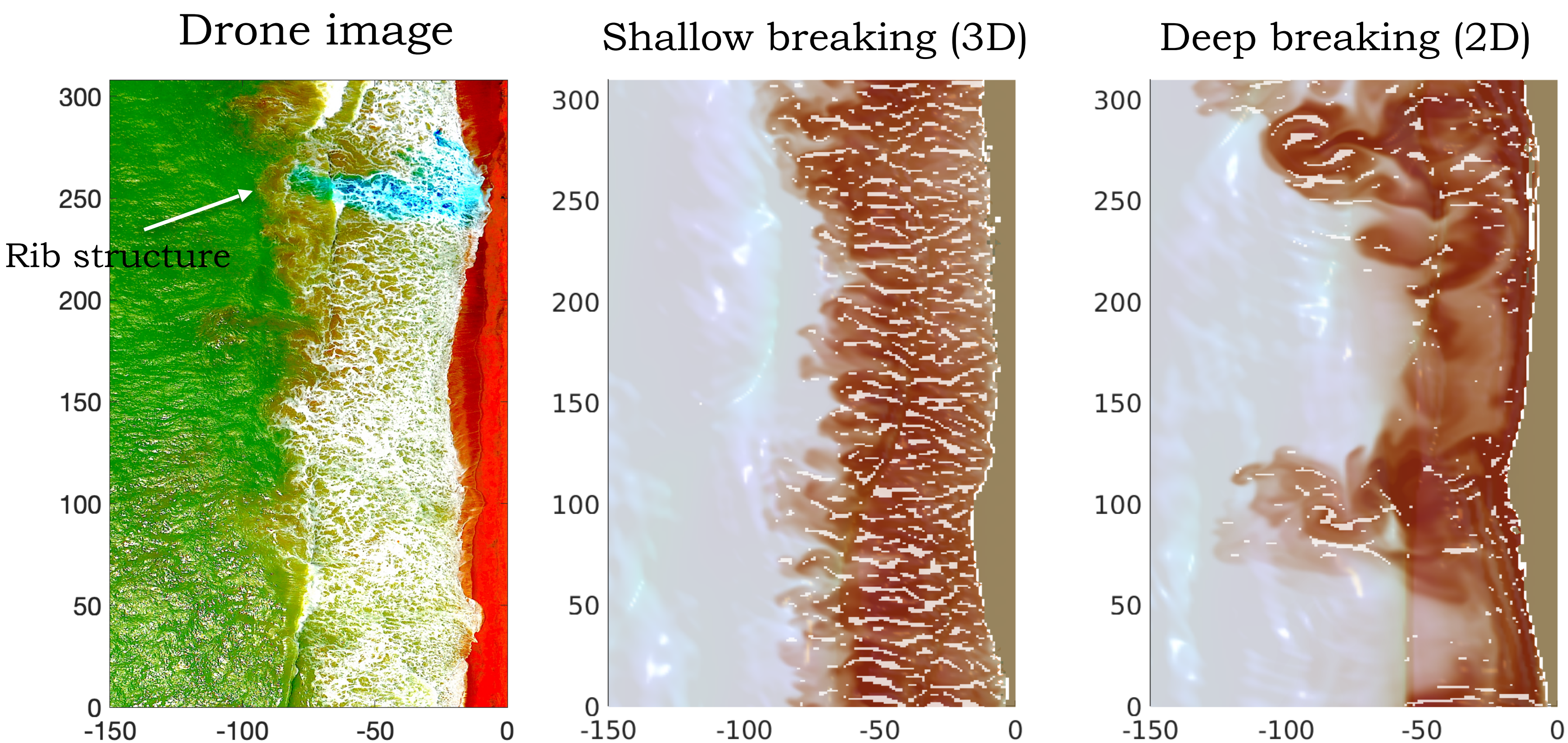
Longshore currents



Wave-mean vertical vorticity patterns
Flash rips and mini-rips



Rib structures in turbidity
with a suspended sediment model

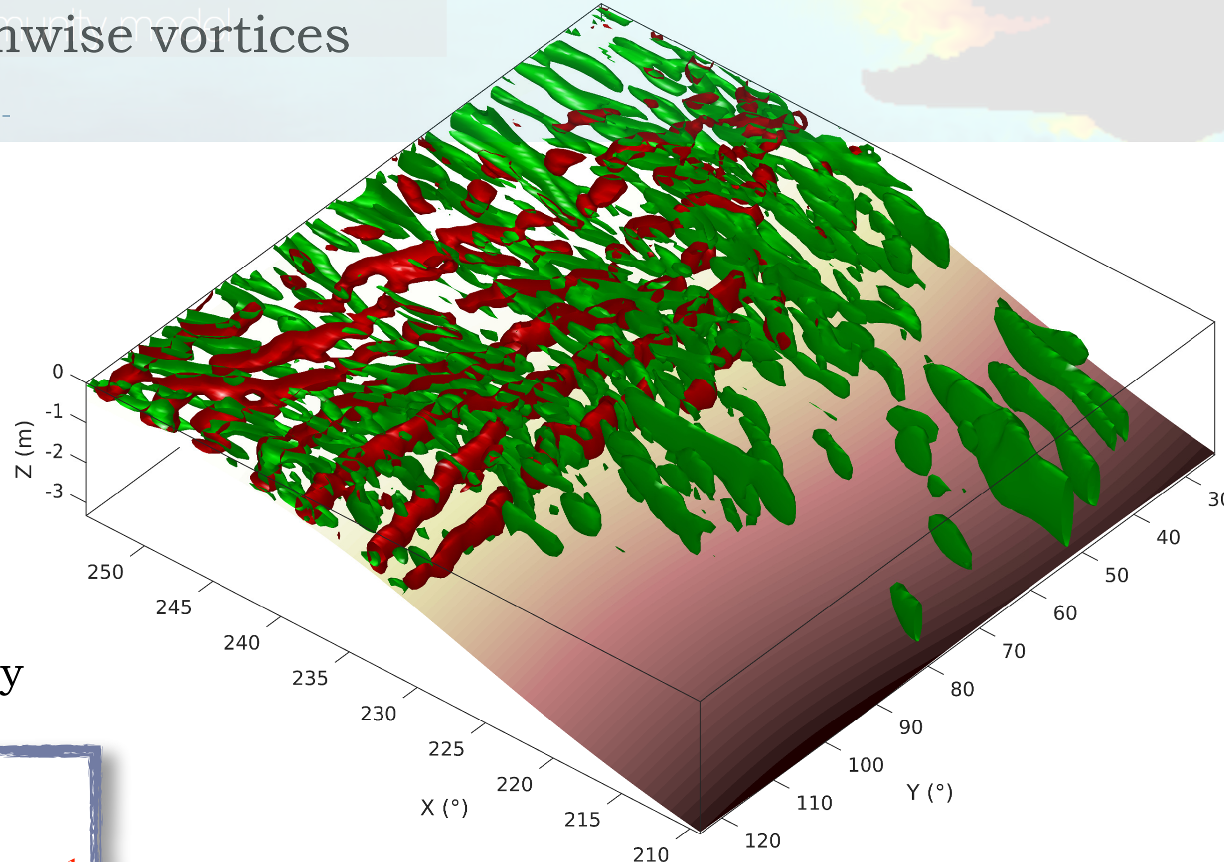


Turbidity patterns (brown) and foam/convergence lines (white)

Vertical Shear instability
spanwise rollers & streamwise vortices

$$Q = -\frac{1}{2}\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}$$

Cross-shore
Alongshore

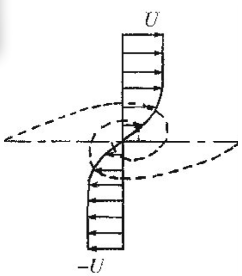


Primary & Secondary instability
(Pierrehumbert & Widnall, 1982):

$$\lambda_{stream\,/\,span} \sim 14\delta \sim 5\ m$$

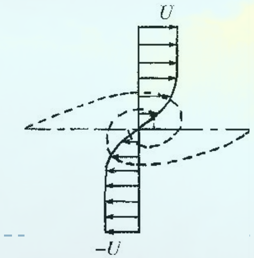
$$f_{stream\,/\,span} \sim 0.015\frac{U}{\delta} \sim (40\ s)^{-1}$$

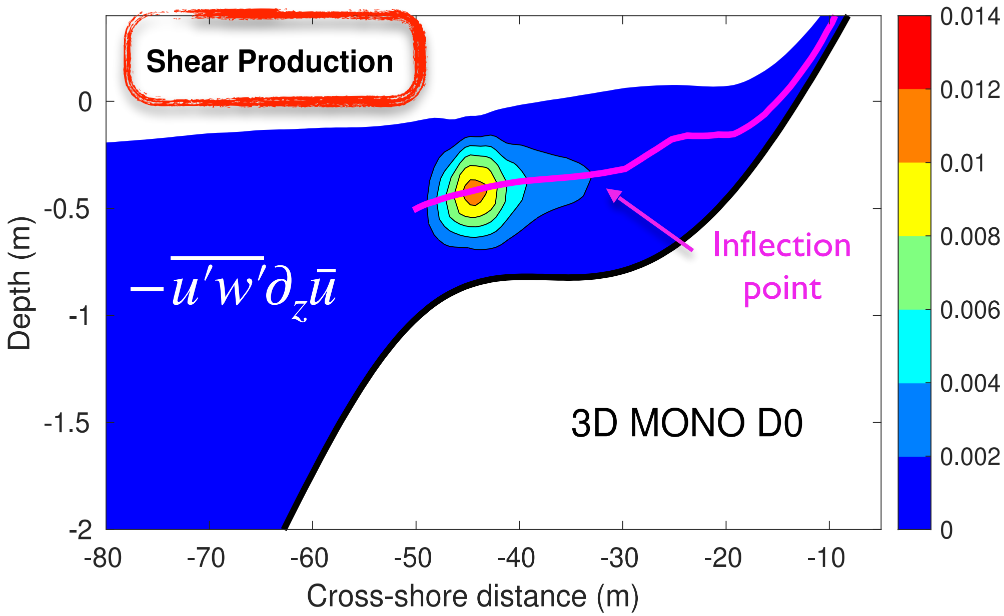
δ is vorticity thickness $\Delta U / \left.\frac{\partial u}{\partial z}\right|_{\max} \sim 40$ cm

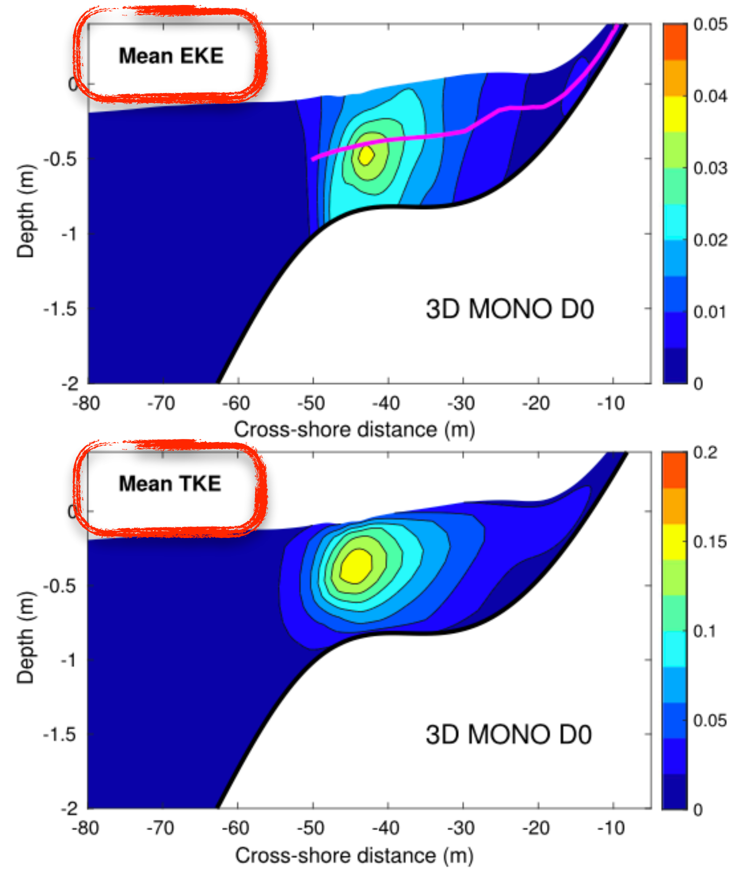


Vertical shear instability

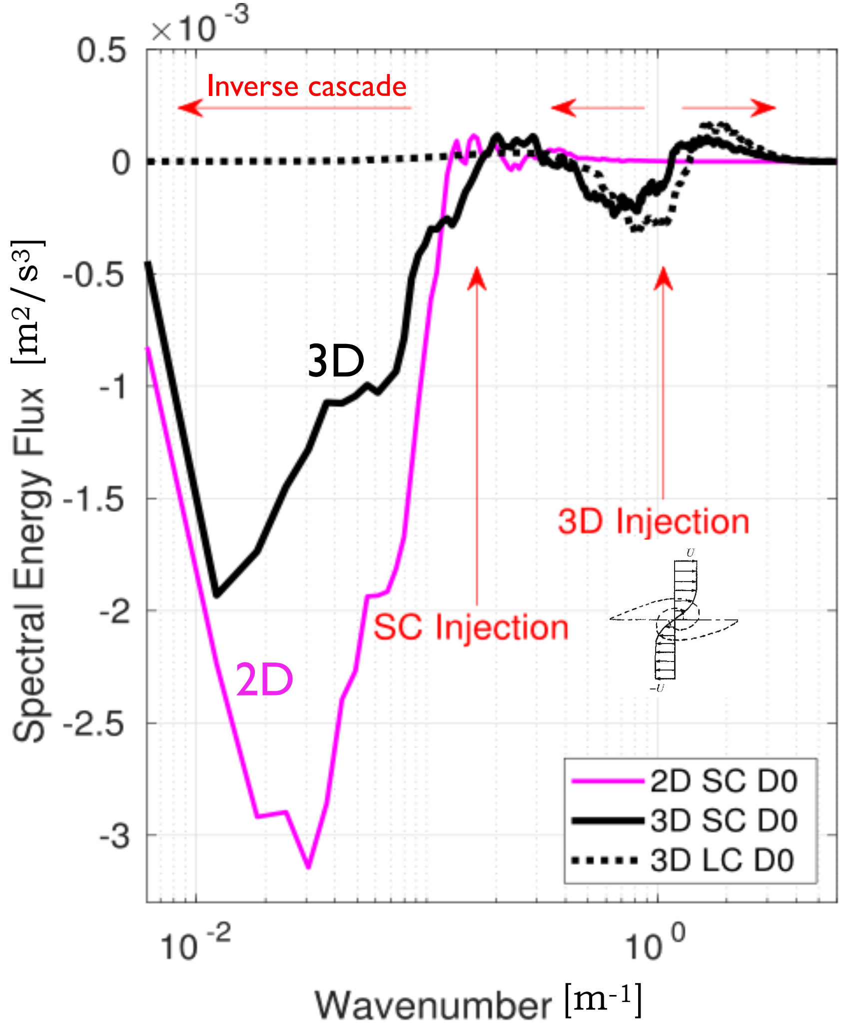
Energy production



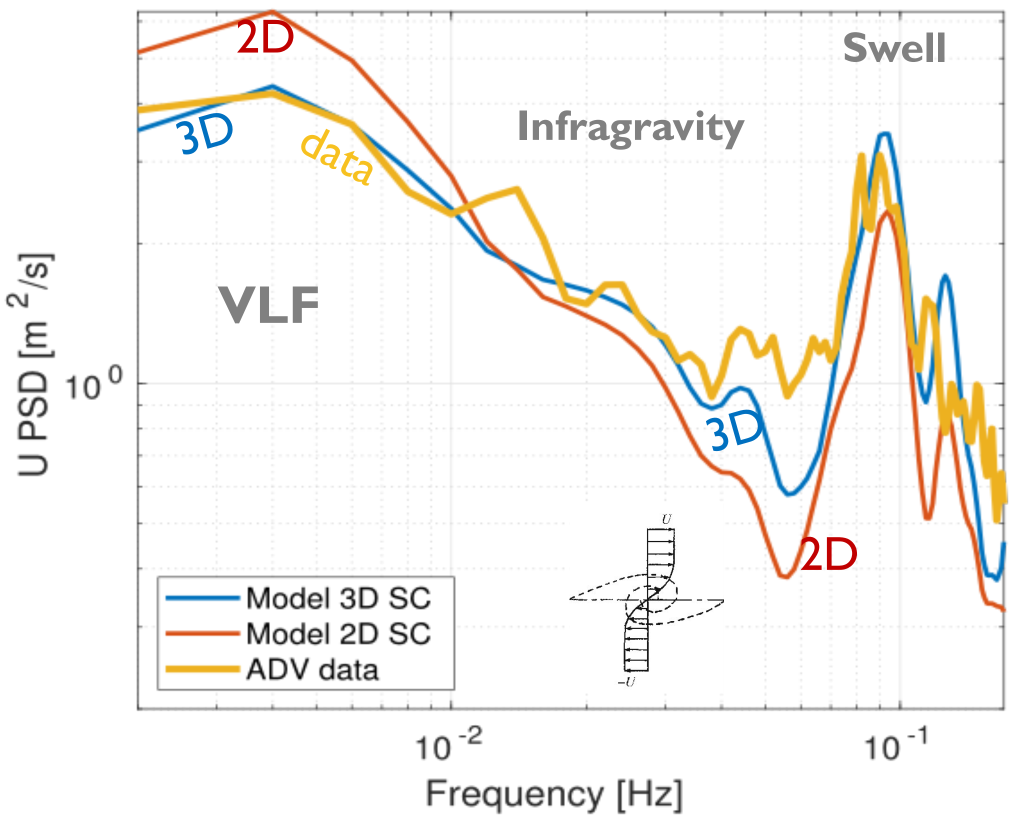




Turbulence cascades
less VLF, more IG eddies



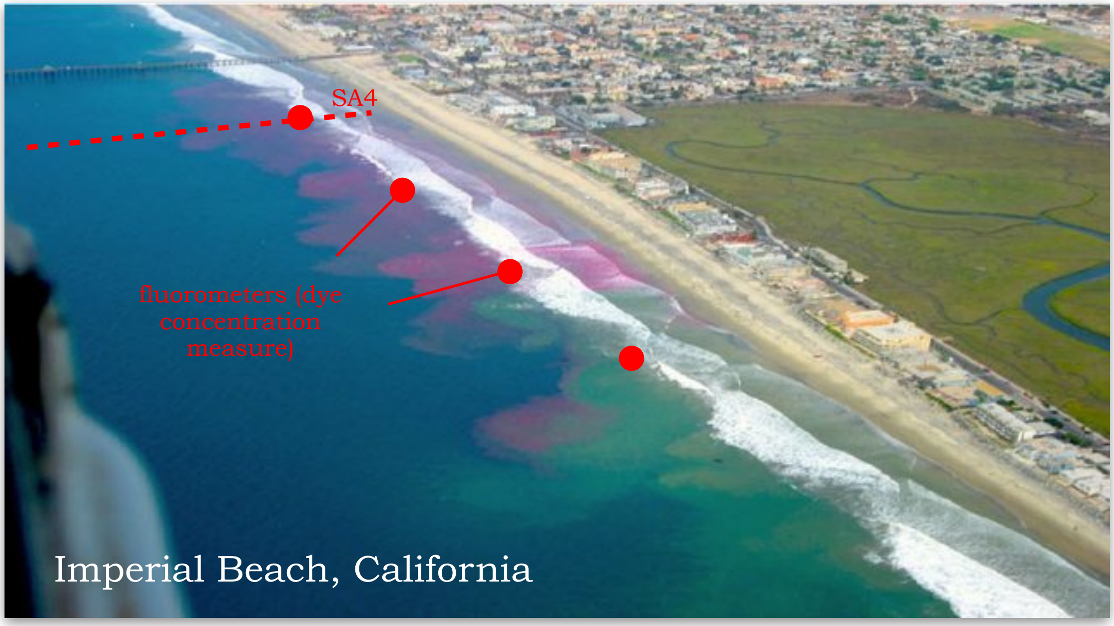
Turbulence cascades
less VLF, more IG eddies



Nearshore mixing

Nearshore mixing
IB09 tracer experiment
PhD thesis of Simon Treillou

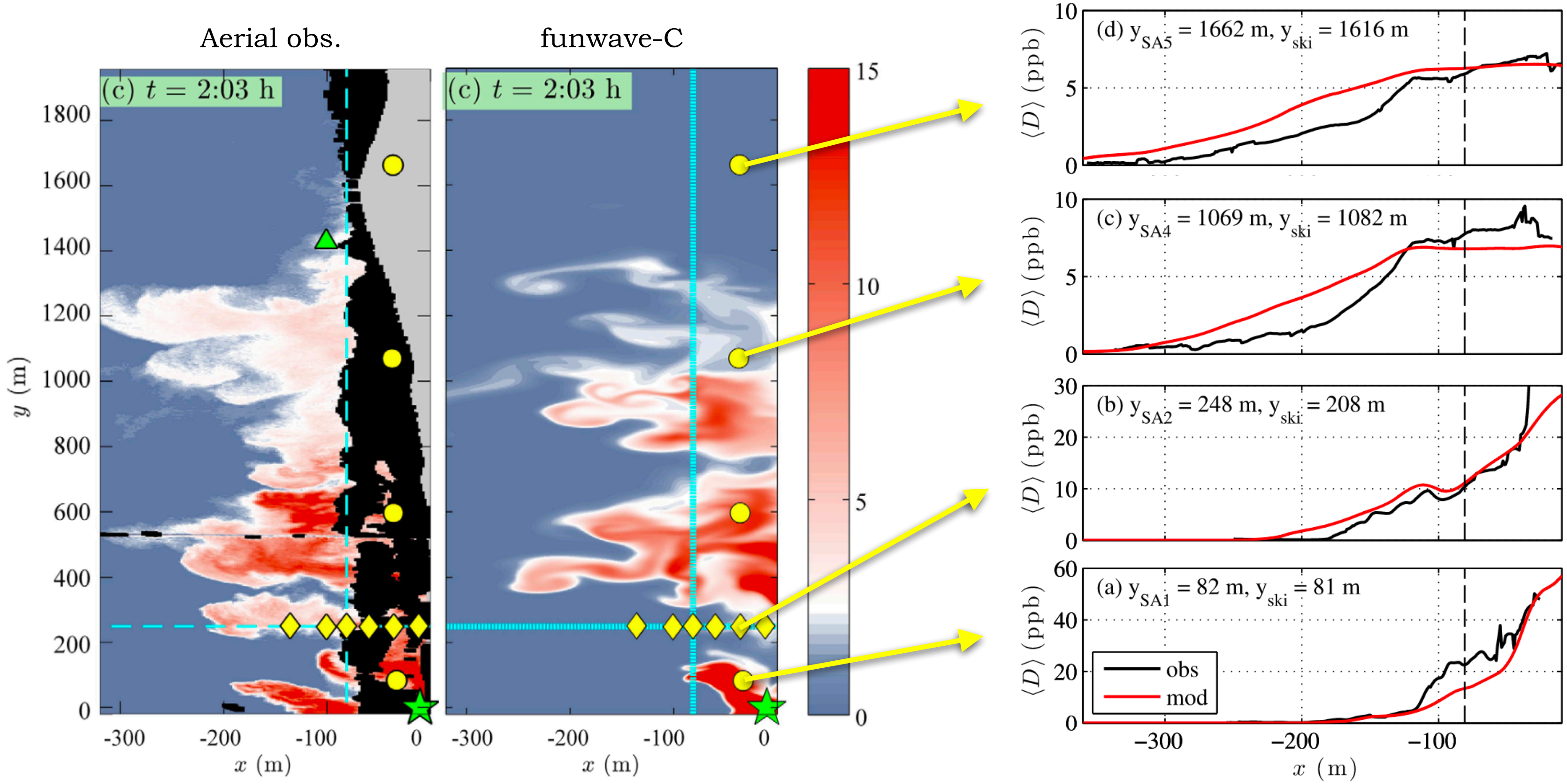
* IB09 experiment for nearshore mixing in the Southern California Bight (Hally-Rosendahl & Feddersen, 2016)



Hally-Rosendahl & Feddersen (2016)

IB09 modeling with Funwave

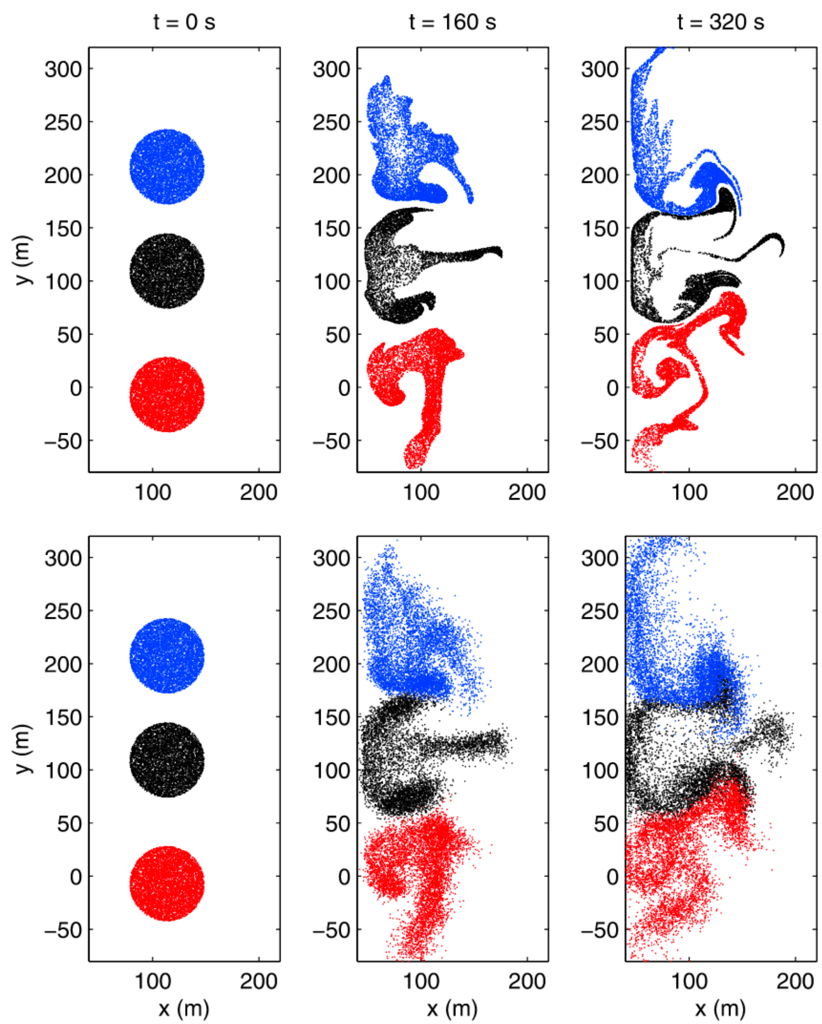
Hally-Rosendahl & Feddersen (2016)



Reasonable results with 2D wave-resolving model, but:
- **underestimates mixing in the surfzone**
- **overestimates exchange with the inner shelf**

a necessary "background" diffusion in 2D models

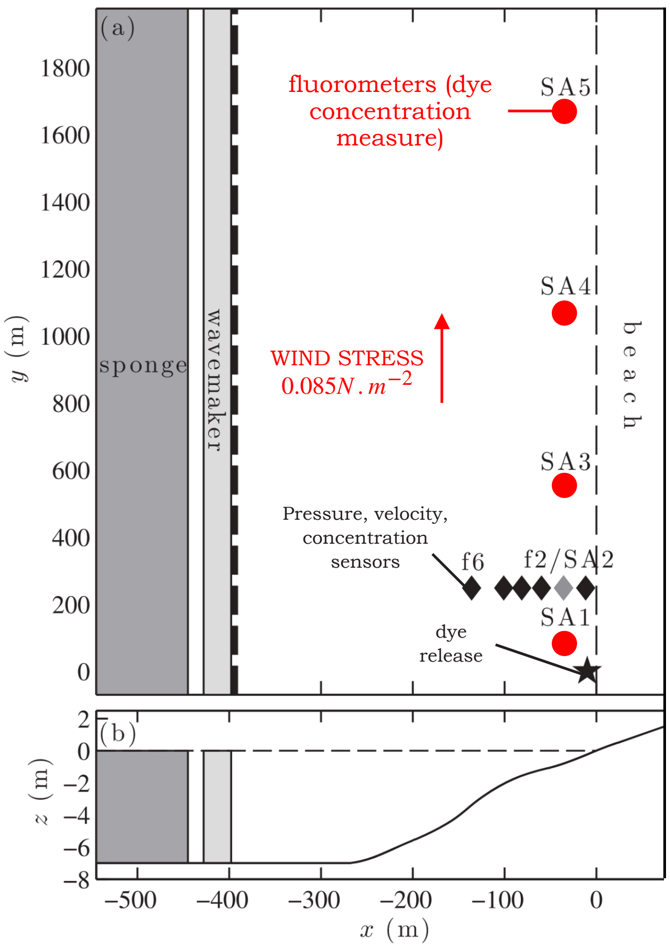
Geiman et al. (2011): model-data comparison of drifter dispersion



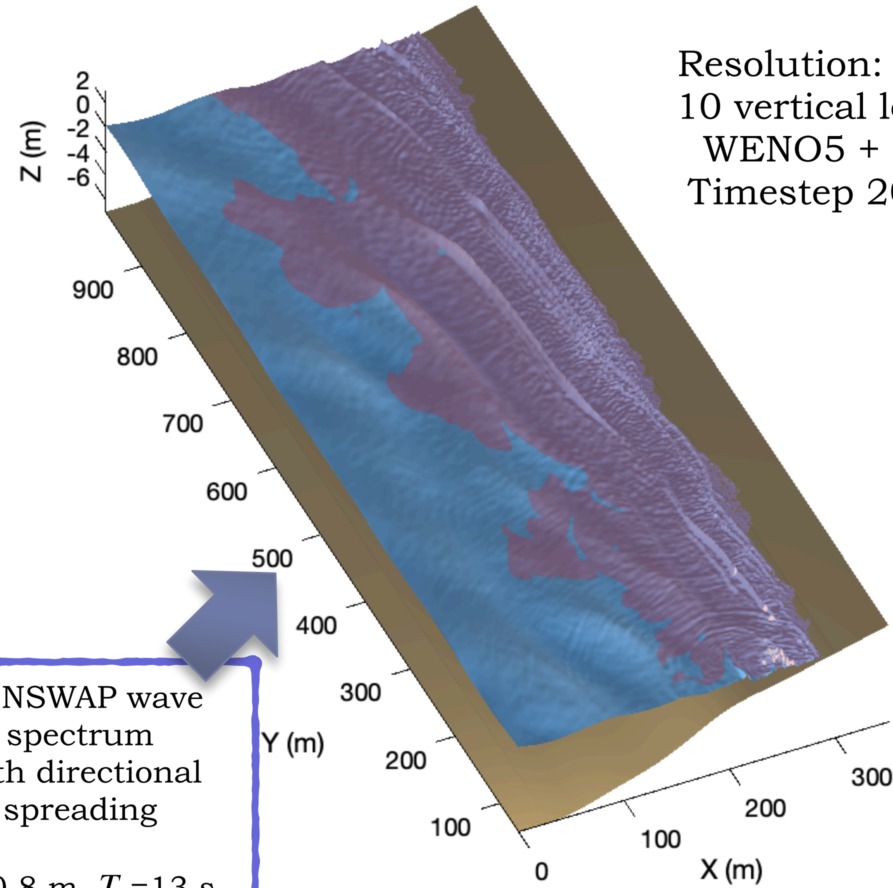
$\kappa_{bg} \sim 0$

$\kappa_{bg} \sim 0.3 \text{ m}^2\text{s}^{-1}$

IB09 CROCO modeling



Modified from Hally-Rosendahl & Feddersen (2016)

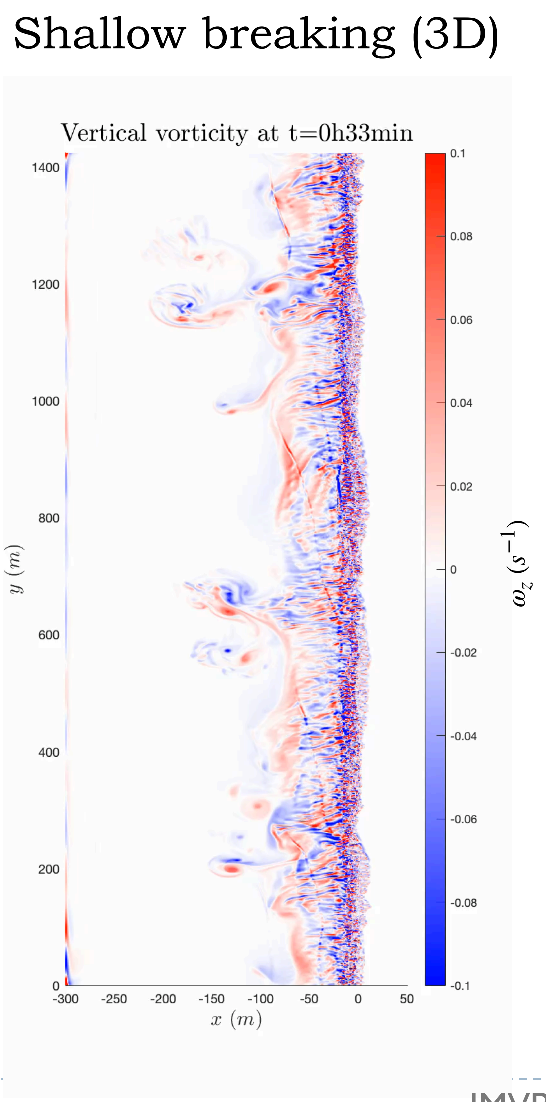


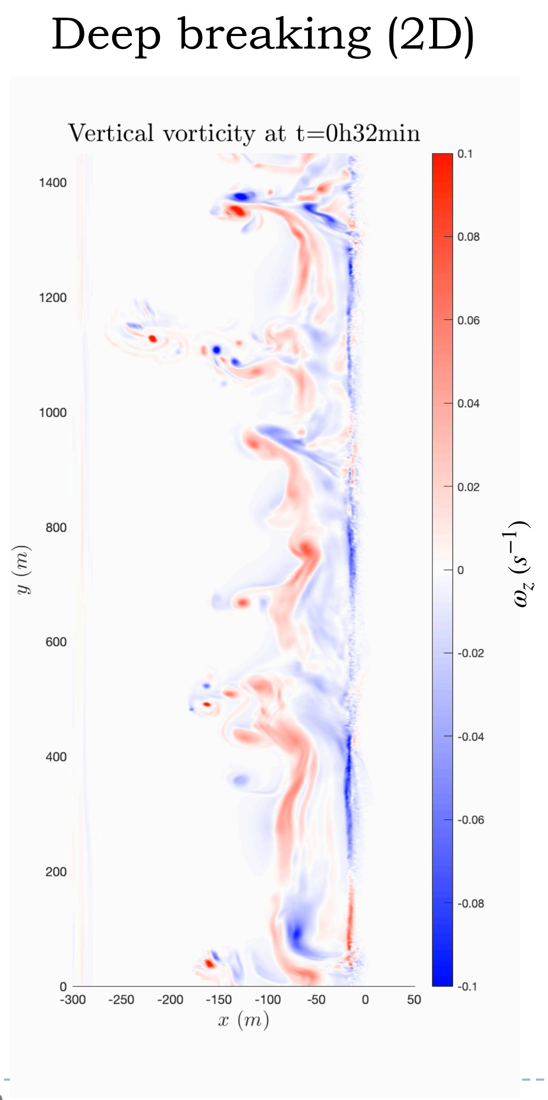
Resolution: 1 m,
10 vertical levels
WENO5 + k-ω
Timestep 20ms

JONSWAP wave spectrum with directional spreading

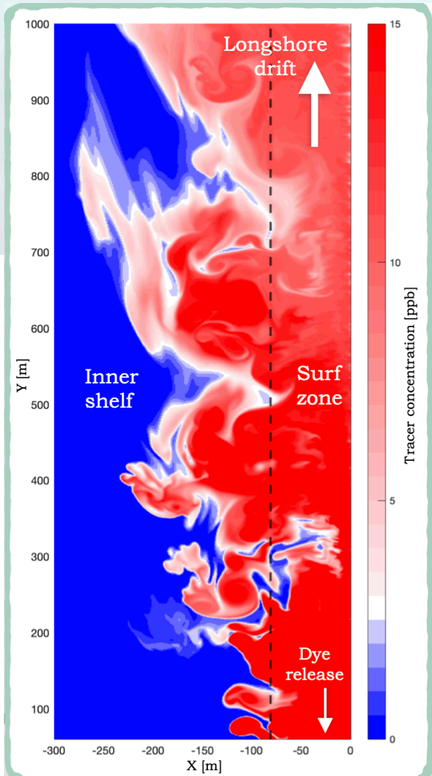
H_s=0.8 m, T_p=13 s,
θ=17°, σ=10°, γ = 20
(October 13 2009)

IB09 CROCO modeling

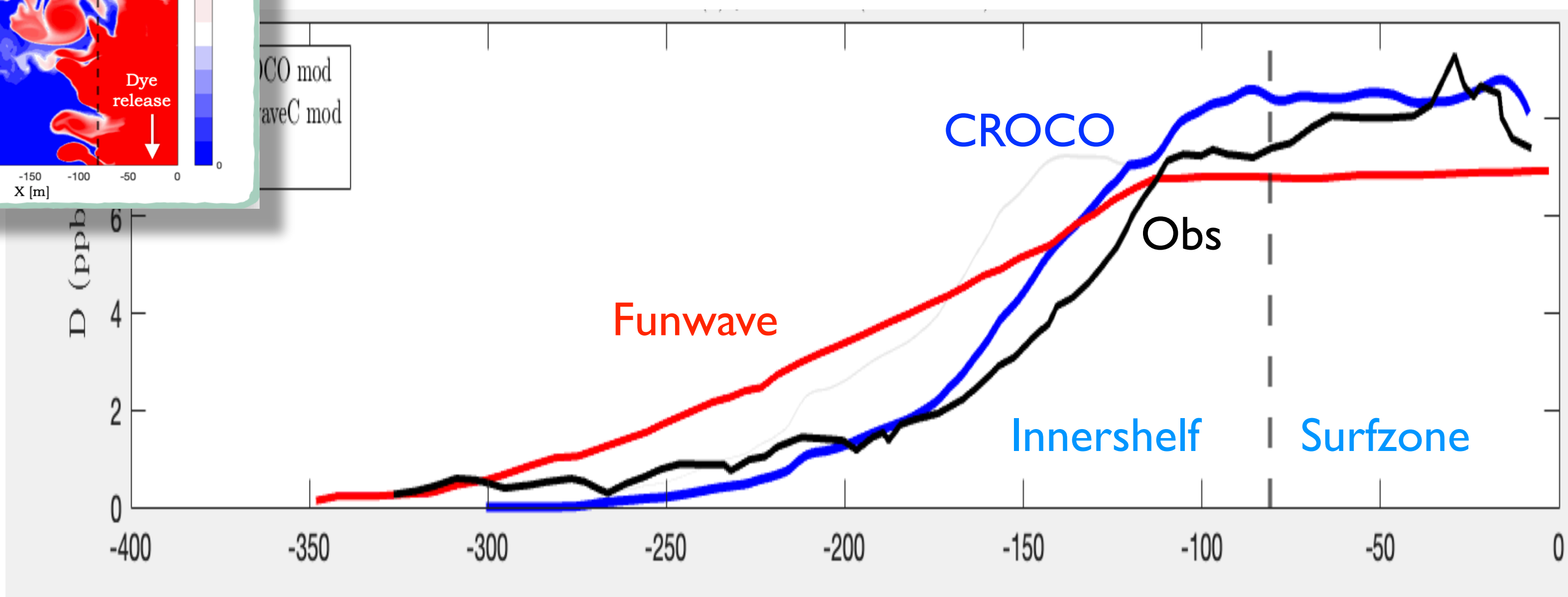




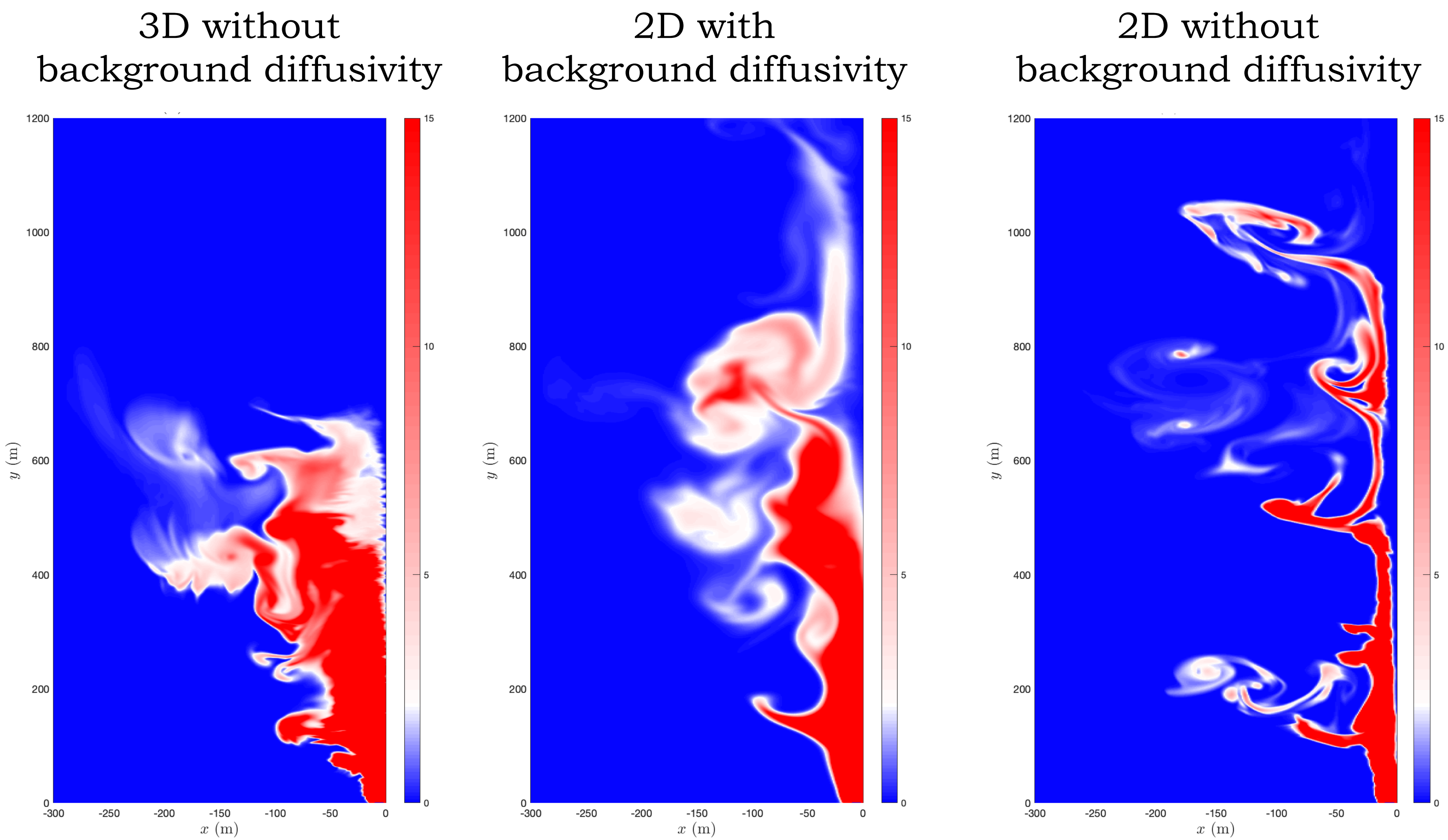
Surf-shelf exchange



Tracer profile at y =1000 m



Surfzone mixing

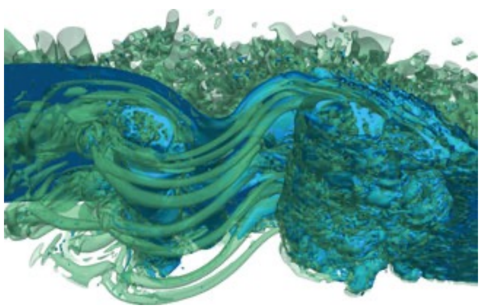


Effective diffusivity

$\kappa_x \sim 1\ \mathrm{m^2 s^{-1}}$ (Clark et al., 2010)

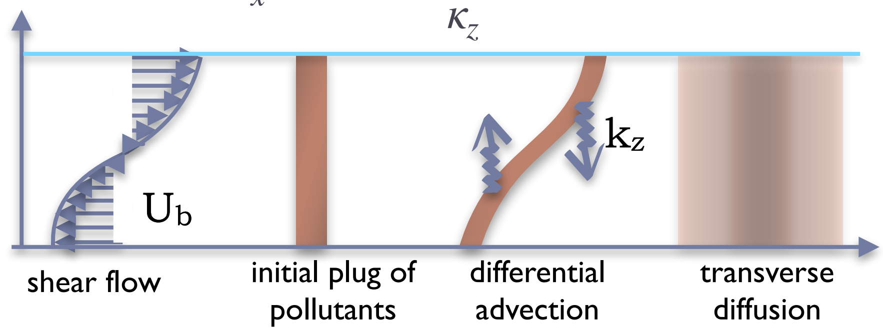
Breaking waves (Svenden et al., 1987)

$$\kappa_x \sim 0.01 \sqrt{g h^3} \sim 0.04\ \mathrm{m^2 s^{-1}}$$



Shear dispersion (Pearson et al., 2009)

$$\kappa_x \sim 0.01 \frac{U_b^2 h^2}{\kappa_z} \sim 0.1\ \mathrm{m^2 s^{-1}}$$

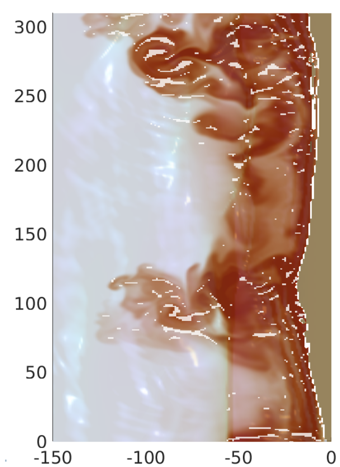


Surf eddies (Clark et al., 2010)

$$\kappa_x \sim 0.1\, U\, L \sim 1\ \mathrm{m^2 s^{-1}}$$

Flash rips

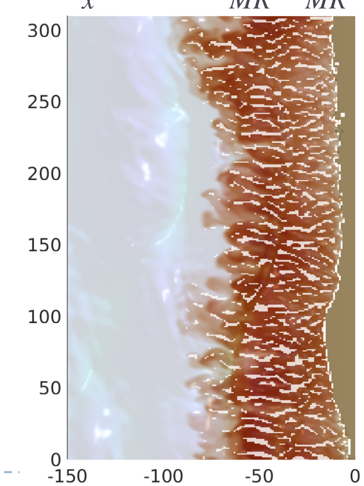
$\kappa_x \sim 0.1\, U_{FR}\, L_{FR}$



OR

Mini-rips

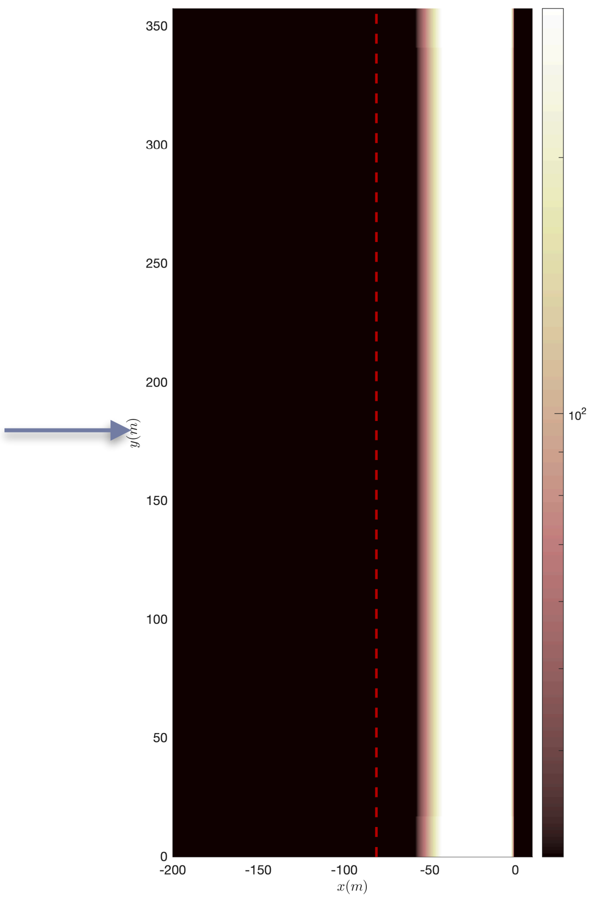
$\kappa_x \sim 0.1\, U_{MR}\, L_{MR}$



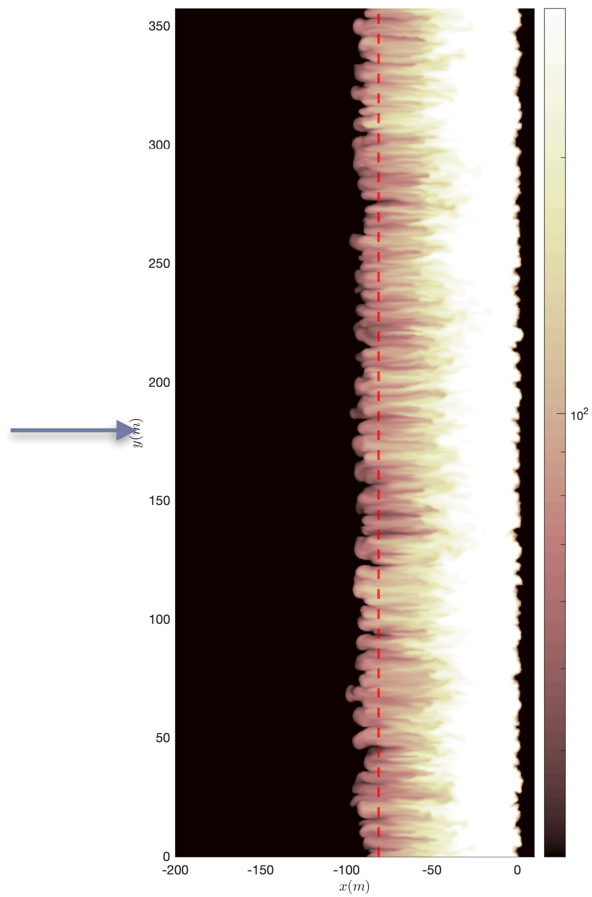
Effective diffusivity

Shear dispersion

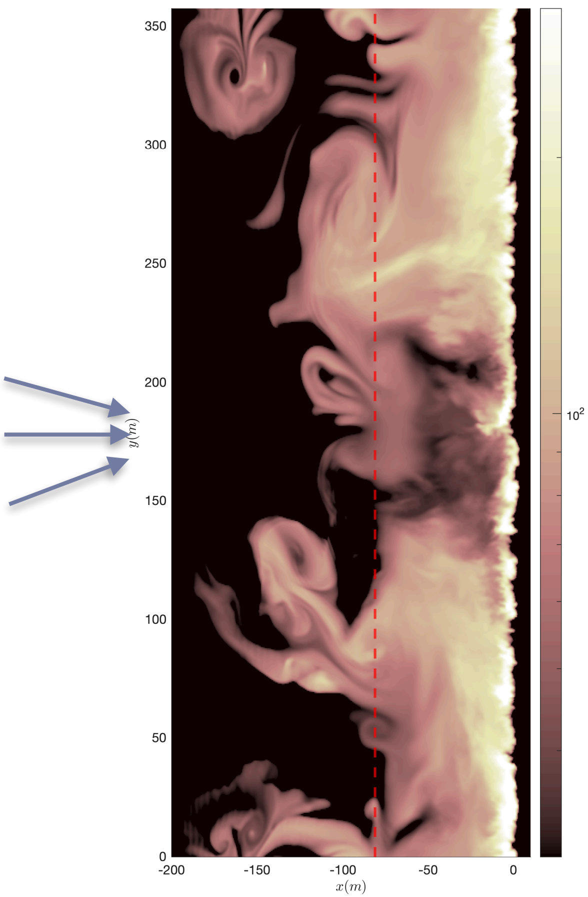
(monochrom. waves no perturbation)



Shear dispersion + mini-rips



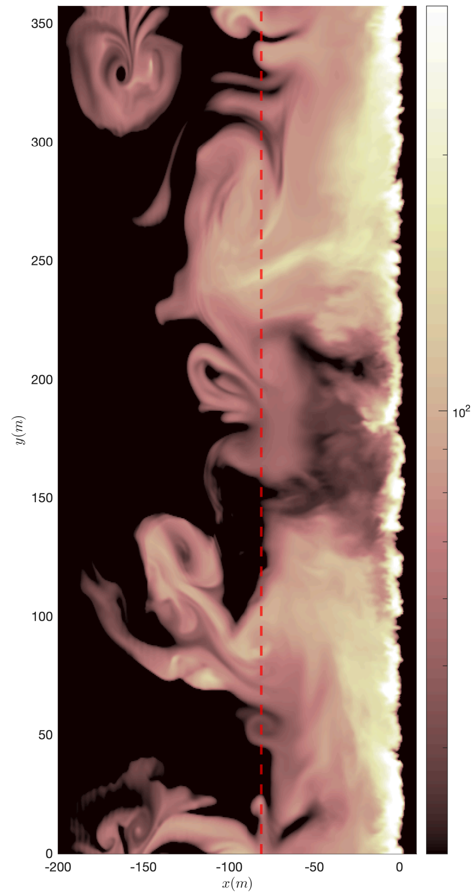
Shear dispersion + mini-rips + Flash rips



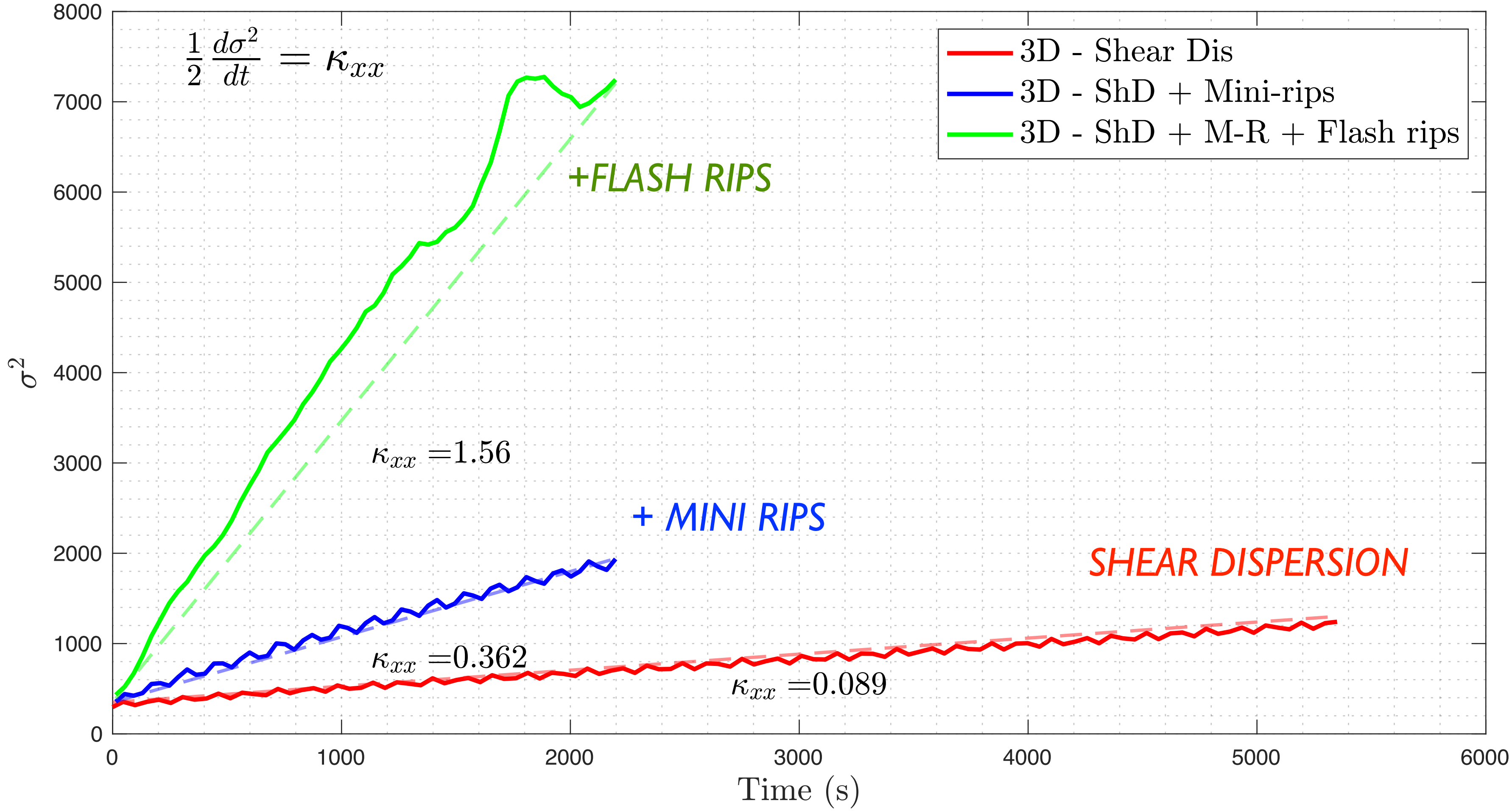
Effective diffusivity

Estimation of surfzone cross-shore diffusivity (Clark et al., 2010):

$$\kappa_{xx} = \frac{1}{2}\frac{d\sigma^2}{dt} \qquad \text{with} \qquad \sigma^2 = \frac{\int_{-x_b}^{0} [x]^2 \bar{D}(x,y)dx}{\int_{-\infty}^{\infty} \bar{D}(x,y)dx}$$



Effective diffusivity



CONCLUSIONS

- **Free-surface, wave-resolving RANS models** are now ready for realistic nearshore problems

- **Vertical shear alters nearshore dynamics**
  - Shear instability produces mini rips: intermediate range of turbulence (within IG range)
  - Reduces inverse cascade toward VLF (flash rips)
  - Reduces instability of longshore drift

- **Impact on nearshore dispersion**
  - Weaker flash rips reduce surf-shelf exchange
  - Shear dispersion and mini-rips provide fast mixing in the surfzone (*background diffusivity*)