Modélisation du déferlement des vagues sur des bathymétries variables 2D et 3D

Marissa L. Yates[1], Sunil Mohanlal[1], Jeffrey Harris[1], Stephane Grilli[2]

[1]LHSV, Ecole des Ponts ParisTech, EDF R&D

[3]Department of Ocean Engineering, University of Rhode Island

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Journées de Modélisation des Vagues à Phases Résolues

Outline

Background and motivation

Mathematical and numerical models

- Wave breaking modeling

2D and 3D test cases

Laboratory experiments

Summary and ongoing work

Background and motivation

Background and motivation

Wave impacts

Coastal risks

Coastal zone wave modeling

- **Objective:** develop an accurate, nonlinear, phase-resolving nearshore wave propagation model
- **Challenge:** accurate and computationally efficient modeling of the dominant physical processes at a wide range of spatial and temporal scales
- **Current Work:** wave breaking effects and extension to 3D

Wave breaking: 3DWaveBI project

Improve modeling of: (1) far-field wave conditions,
(2) **wave breaking**,
(3) wave forces on structures

Breaking waves

Spilling breaker

Plunging breaker

Surging breaker

Importance of modeling wave breaking:

- offshore and coastal wave forecasting
- estimating wave forces on coastal and maritime structures
- evaluating air-sea gas and heat exchanges

Breaking waves

Spilling breaker

Plunging breaker

Steepness-limited
(deep water)

Breaking waves

Spilling breaker
$\xi_0 < 0.5$

Plunging breaker
$0.5 < \xi_0 < 3.3$

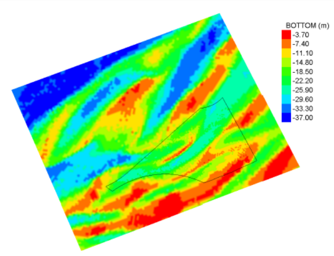
Surging breaker
$3.3 < \xi_0$

Steepness-limited	Depth-limited
(deep water)	(shallow water)

where $\xi_0 = \dfrac{m}{\sqrt{H_0/L_0}}$

Wave breaking statistics

- Where do waves break?
- What forces do breaking waves generate on structures?
- What type of wave breaking?



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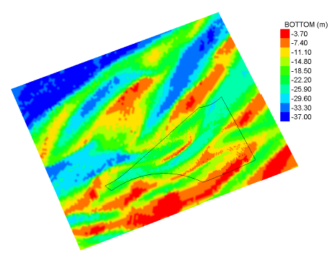
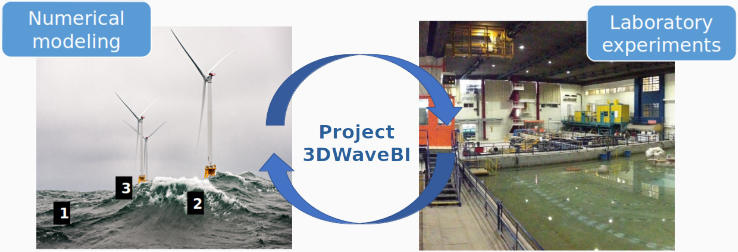
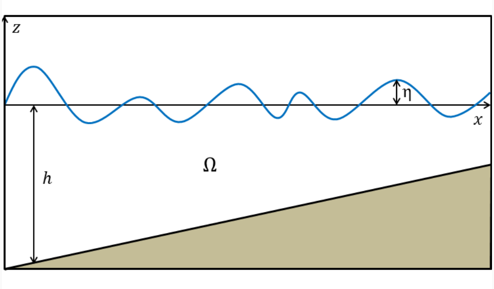




Mathematical and numerical models

Mathematical model



- incompressible flow
- inviscid fluid
- homogeneous atmospheric pressure
- irrotational (potential) flow $\nabla\phi = \underline{u}(x, z, t)$

Water wave problem

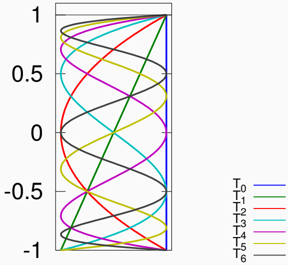
1. Laplace equation $\nabla^2\phi = 0$ in Ω
2. KFSBC (no flow across interface)
3. DFSBC (Bernoulli equation)
4. Bottom and lateral boundary conditions

Numerical model

Misthyc code

Calculating the free surface velocity potential

- Horizontal resolution: high order finite difference method
  (e.g. Bingham et al., 2007)
- Vertical resolution: spectral method
  (Tian et al., 2008)



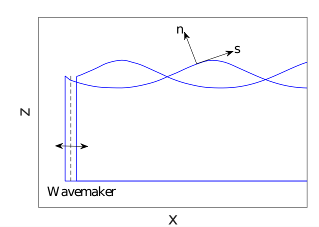
Advancing in time

- Zakharov equations:
  $$\eta_t = -\nabla\eta\nabla\tilde{\phi} + \tilde{w}\left(1 + (\nabla\eta)^2\right)$$
  $$\tilde{\phi}_t = -g\eta - \frac{1}{2}(\nabla\tilde{\phi})^2 + \frac{1}{2}\tilde{w}^2\left(1 + (\nabla\eta)^2\right) \text{ with } \tilde{w} = \left.\frac{\partial\phi}{\partial z}\right|_{z=\eta}$$
- Temporal integration with explicit 4th order Runge-Kutta method

Yates and Benoit (2015), Raoult et al. (2016), Benoit et al. (2017)

Numerical model

NWT (Numerical Wave Tank) code



Calculating the free surface velocity potential

- Boundary Integral Equation
  $$\alpha(\mathbf{x}_i)\phi(\mathbf{x}_i) = \int_\Gamma \left\{ \frac{\partial \phi}{\partial n}(\mathbf{x}) G(\mathbf{x}-\mathbf{x}_i) - \phi(\mathbf{x}) \frac{\partial G}{\partial n}(\mathbf{x}-\mathbf{x}_i) \right\} d\Gamma$$
  G - Green's function for Laplacian

Advancing in time

- Mixed Eulerian-Lagrangian frame of reference
  $$\frac{D\mathbf{r}}{Dt} = \frac{\partial \mathbf{r}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{r} = \mathbf{u} = \nabla\phi$$
  $$\frac{D\phi}{Dt} = -gz + \frac{1}{2}|\nabla\phi|^2 - p_a$$
- Temporal integration with 2^{nd} order Taylor series expansion

Grilli et al. (1989)

Wave breaking

How can the effects of wave breaking be modeled?

1. Wave breaking initiation

2. Energy dissipation

3. Wave breaking termination

Seeking a unified approach from deep to shallow water:
Is this possible?

Types of wave breaking criteria

- **Geometric criteria:** based on the geometric characteristics of the wave (e.g. steepness, horizontal asymmetry, angle of wave front)
  (e.g. Rapp and Melville, 1990; Schäffer et al., 1993)

- **Kinematic criteria:** when the fluid velocity exceeds the speed of wave propagation ($U/C > 1$)
  (e.g. Kennedy et al., 2000; Stansel and Farlane, 2002; Tian et al., 2010; D'Alessandro and Tomasicchio, 2008)

- **Dynamic criteria:** when the local wave energy flux exceeds a threshold: $B_x = \frac{F_x}{Ec_x} = U_x/C_x$
  (e.g. Barthelémy et al., 2018)

Wave energy dissipation mechanisms

- **Hydraulic jump model:** analogy between breaking waves and hydraulic jump *(e.g. Guignard and Grilli, 2001)*
- **Eddy viscosity model:** dissipating energy with an eddy viscosity *(e.g. Kennedy et al., 2000; Kurnia and van Groesen, 2014)*
    - Vorticity model: separating the flow into the irrotational and rotational components and resolving a vorticity transport equation *(e.g. Svendsen et al., 1996; Veeramony and Svendsen, 1998)*
    - TKE closure model: solving a PDE estimate the eddy viscosity as a function of the turbulent kinetic energy *(e.g. Zhang et al., 2014)*
- **Hybrid model:** turning off the dispersion terms (switching from non-hydrostatic to hydrostatic equations) *(e.g. Tonelli and Petti, 2012; Tissier et al., 2012)*

Wave breaking

How to take into account the effects of wave breaking?

1. Wave breaking initiation → **threshold B=0.85**
   (Barthelemy et al. , 2018; Derakhti et al., 2020)

2. Energy dissipation → **analogy to hydraulic jump**
   (Guignard et Grilli, 2001; Grilli et al., 2019)

3. Wave breaking termination → **termination criterion calibrated for each test case**

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Wave breaking

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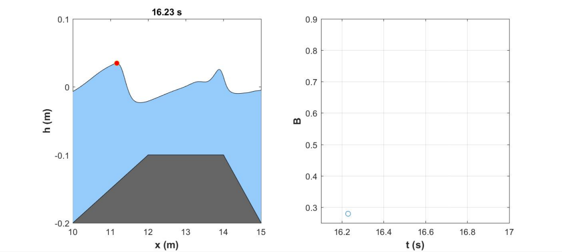
Goal: unified theory of breaking onset and dissipation:

- depth-limited waves (Mohanlal et al., 2023)
- steepness-limited waves (Mohanlal et al., 2022, ICCE)
- depth-limited waves in 3D (Mohanlal et al., submitted)

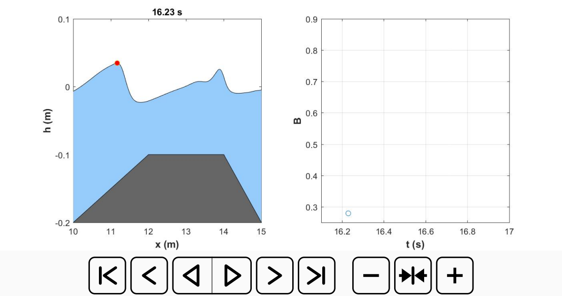
Breaking strategy

Breaking onset



Breaking strategy

Breaking onset



Pressure-type disspiation

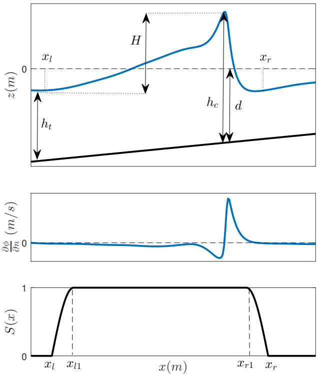
- DFSBC: $\frac{D\phi}{Dt} = -gz + \frac{1}{2}|\nabla\phi|^2 - \boldsymbol{P_a}/\rho$
- $\boldsymbol{P_a}$ = damping pressure

Breaking strength

- $\gamma = T_b \frac{dB}{dt}|_{B=B_{th}}$
- $T_b \equiv \mathrm{T}(x^*, t^*)$

(Derakhti et al. 2018)

Wave breaking dissipation



Hydraulic jump model

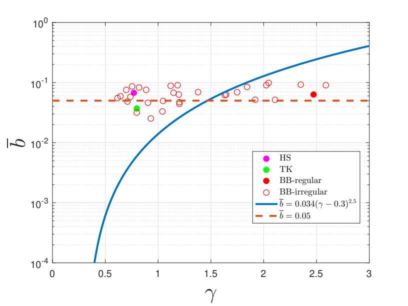
- Instantaneous power dissipated,
  $\Pi(t) = \mu g c d \dfrac{H^3}{4 h_c h_t}$
- $\mu = 1.5$ (Svendsen et al., 1978)

Damping pressure

- Applied for $x \in (x_l, x_r)$
- $\Pi(t) = \int_x P_a(x,t)\phi_n(x,t)\sqrt{1+\eta_x^2}dx$
- $P_a(x,t) = \dfrac{\Pi(t)S(x)\phi_n(x,t)}{\int_x S(x)\phi_n^2\sqrt{1+\eta_x^2}dx}$

Guignard et Grilli (2001); Grilli et al. (2019)

Parameterization



- HS - Hansen Svendsen 1979
- TK - Ting Kirby 1994
- BB - Beji Battjes 1993

(Also previously validated in Papoutsellis et al. 2019, Simon et al. 2019 and Grilli et al. 2020)

Dissipation strength

- In analogy to Duncan (1983):
  $$\overline{b} = \frac{\overline{\Pi} \cdot g}{C_b^5}$$
- $C_b \equiv C(x^*, t^*)$

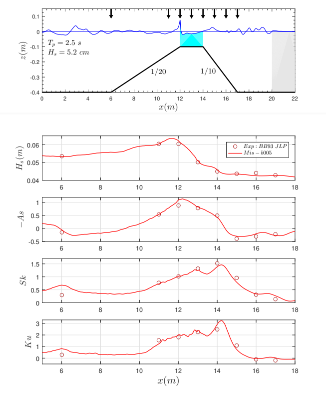
Following scaling law bounds:
let $\overline{b} = 0.05$ for depth-limited breaking

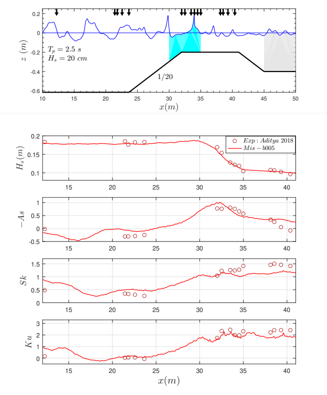
- extended to 3D along quasi-uniform 2D sections of wave crests (Mohanlal et al., submitted)

2D and 3D test cases

Wave statistics : 2D irregular wave breaking

Misthyc model



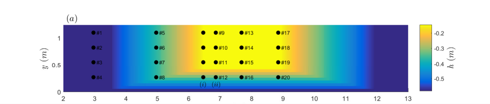


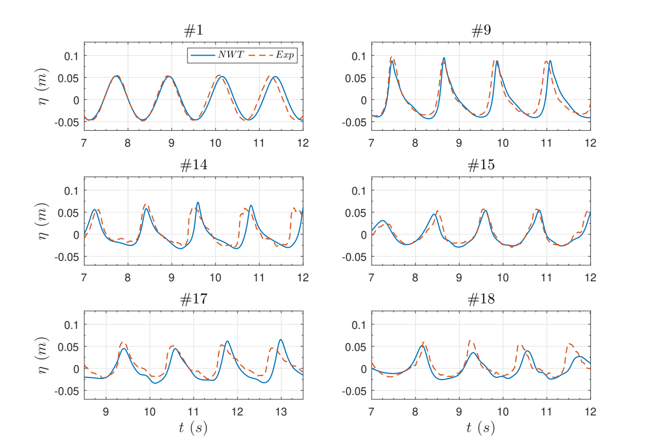
(Beji and Battjes, 1993; Adytia et al., 2018)

Mohanlal et al., 2023

Wave statistics : 3D regular wave breaking

NWT model



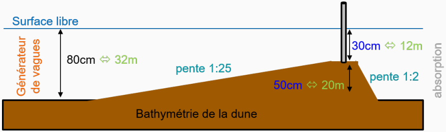


(Kamath et al., 2022)

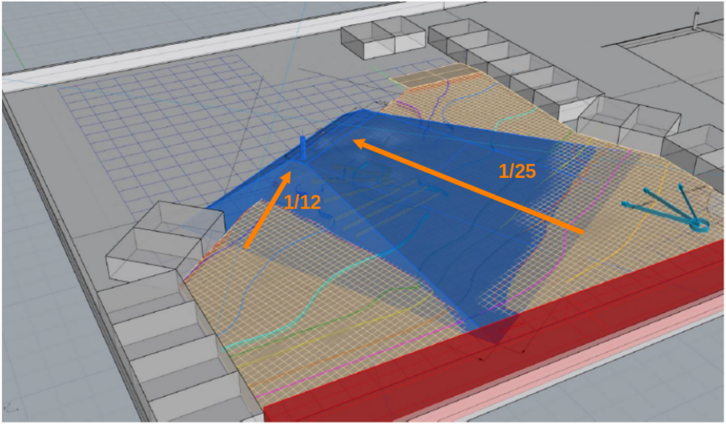
Mohanlal et al., submitted

Laboratory experiments

3D wave tank



Surface libre
Générateur de vagues
80cm ⇔ 32m
pente 1:25
30cm ⇔ 12m
50cm ⇔ 20m
pente 1:2
absorption
Bathymétrie de la dune



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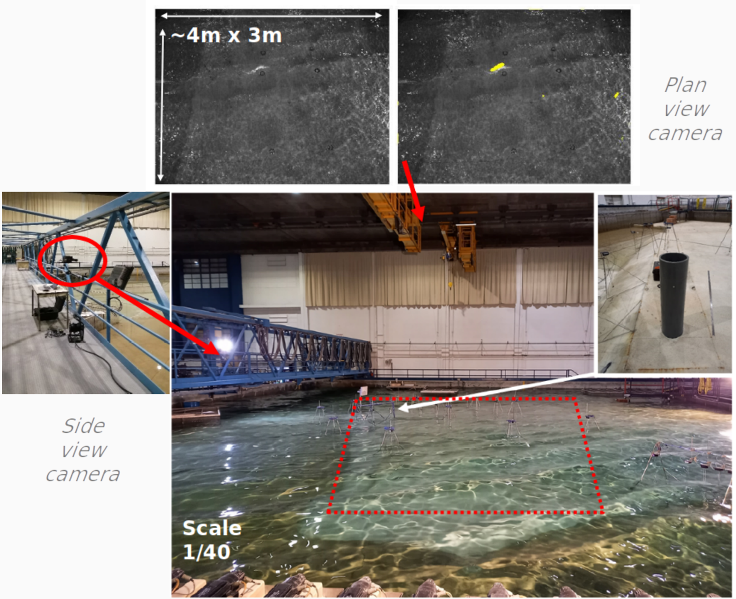
Measurements

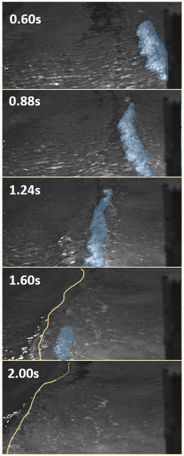


Measurements

Top view

Identification of wave breaking zones
(Internship G. Dreysse)

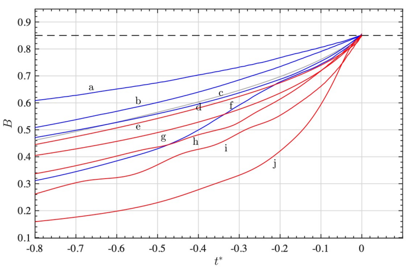
Side view



Characterization of breaking waves
(Internship A. Guidal)

Summary and ongoing work

Summary



Depth-limited breaking

- verified using $B = 0.85$
- proposed $b = 0.05$
- preliminary work in extending to 3D is promising
- steepness-limited breaking uses variable b

Ongoing work

- investigating ξ_0 (red=plunging, blue=spilling)
- validating the 3D model
- comparing the 2D and 3D simulations in the planned laboratory experiments
- ... laboratory experiments...

Thank you!

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